AN ELEMENTARY MANUAL OF STATISTICS

FOURTH EDITION

ELEMENTS OF STATISTICS

BY

A. L. BOWLEY, Sc.D.

PROFESSOR OF STATISTICS IN THE UNIVERSITY OF LONDON

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AN ELEMENTARY MANUAL OF STATISTICS

BY

ARTHUR L. BOWLEY, Sc.D., F.B.A.

PROFESSOR OF STATISTICS IN THE UNIVERSITY OF LONDON

AUTHOR OF "ELEMENTS OF STATISTICS," "NATIONAL PROGRESS SINCE 1882," ETC.

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PREFACE

This manual is intended for the use of those who desire some knowledge of statistical methods and statistical results without going deeply into technicalities or undertaking mathematical analysis; it is hoped that it will be of service to all who have occasion to use statistics in their own business or profession, or who take an intelligent interest in public affairs.

It is also designed as a first course in statistics for students who wish to proceed further in the subject and, if it serves its purpose will stimulate interest in the many fascinating problems that await solution, and that can only be attacked by the methods of modern mathematical statistics.

The first part deals with elementary methods and with such technical terms and ideas as are indispensable in the handling of numbers on a large scale. In the second part the origin of many groups of public statistics is shown, their adequacy is criticized, and some of the more interesting results which are based on them are briefly summarized. This part is intended as a guide to official statistics, not as a compendium or dictionary of them; and the problems attacked are given rather as illustrations than as substantial contributions to knowledge.

To facilitate the use of the book in the hands of teachers a number of exercises of various degrees of complexity are given in Appendix I. Every serious student of commercial or public affairs should be acquainted with the nature of the contents of the Statistical Abstract of the United Kingdom; to promote this knowledge, and because many pages of headlines and figures would otherwise have been necessary, a large proportion of the examples relate to tables in the Abstract for 1909,* for which future or earlier abstracts can readily be substituted.

Appendix II contains a short list of Blue Books which should be easily accessible in the Library of every institution where the subject of statistics has a place in the curriculum.

My thanks are due to Dr. Dudfield (Medical Officer of Health for Paddington) and Professor Cannan for most useful criticism of some of the chapters, and to Mr. G. W. Palmer for help and advice in the correction of proofs. I shall be grateful for any criticisms which will tend to increase the utility or improve the accuracy of the book.

Reading,
December 1909.

A. L. B.

SECOND EDITION

The tables have, so far as possible, been continued so as to include statistics for 1913; those for 1914 would be hardly comparable with former years without much explanation. Minor corrections and alterations have been made and notes added in the text or at the end, when information of new kinds was available. For many of these I am indebted to Miss E. Dutt.

July 1915.

* 1914 in second edition.

CONTENTS

PART I

CHAP.		PAGE
I.	NATURE AND USE OF STATISTICS	1
II.	ACCURACY AND APPROXIMATION	6
III.	AVERAGES	15
IV.	THE ACCURACY OF AVERAGING AND OTHER ARITHMETICAL PROCESSES	27
V.	USE OF DIAGRAMS	35
VI.	TABULATION	50
VII.	SAMPLING	56
VIII.	RULES FOR USING PUBLISHED STATISTICS	64
IX.	METHODS OF STATISTICAL ANALYSIS	71

PART II

I.	THE POPULATION CENSUS	85
II.	VITAL STATISTICS	102
III.	TRADE AND TRANSPORT	111
IV.	PRICES	126
V.	PRODUCTION	133
VI.	WAGES	137
VII.	EMPLOYMENT	151
VIII.	OTHER WORKING-CLASS STATISTICS	161
IX.	INCOME AND CAPITAL	171
X.	TAXES AND RATES	185

APPENDIX I.	EXERCISES	197
II.	SELECTED LIST OF BOOKS OF REFERENCE	215
III.	THE POPULATION CENSUS OF 1911	217
IV.	REVENUE AND EXPENDITURE, 1908–1913	222
	INDEX	223

PART I

NOW READY

A KEY

(With Explanatory Notes)

to the Exercises and Questions in

AN ELEMENTARY MANUAL OF STATISTICS

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AN ELEMENTARY MANUAL OF STATISTICS

CHAPTER I

NATURE AND USE OF STATISTICS

1. Statistics are numerical statements of facts in any department of inquiry, placed in relation to each other; statistical methods are devices for abbreviating and classifying the statements and making clear the relations. The elementary methods are based on arithmetical processes of an easy but specialized kind; more refined methods, necessary for certain classes of investigation, involve complex mathematical ideas.

2. Statistical treatment is necessary in a very great variety of cases, some of which may be distinguished as follows—

Groups.—If a large number of things or persons have something in common, *e.g.* as members of the same nation, workers in the same occupation, houses in a defined locality, but differ one from the other in respect to some measurable characteristic, *e.g.* age, amount of wages, rateable value, together they form a statistical group. Groups can be represented by *diagrams*, *tabulated* in grades, or described in abbreviated form by *averages*.

Classes.—If the characteristics in which the things or persons differ are not measurable, but need separate description, *e.g.* the number of persons in different districts, or in different occupations in the same industry, or of houses used for

different purposes, a statistical *table* can be made showing in juxtaposition the numbers in various classes and sections according to any scheme of classification, and the relative sizes of the classes can be indicated by *percentages*.

Series.—If the numbers in some group or class are counted, or the quantities or values of some aggregate are measured, periodically (weekly, monthly or annually), we obtain a statistical series, whose nature is most easily appreciated with the help of a diagram.

3. Statistics are thus used for describing and analyzing large groups or aggregates, too large or complex to be intelligible by simple observation. Thus the affairs of a community, the progress of a large business, and the productivity of a country need statistical treatment, while the individual, the single transaction, the quantity grown in a field do not. The difference is not one of degree only, for when investigation is extended over a large area, regularity is obtained, conformity to general laws is visible, and new methods of description are required, while observation of a few cases suggests only chance and chaos. There is infinite variety in the constitution of a family, but in a community the distribution by age is nearly invariable. Men differ from each other in stature and in wealth; but simple mathematical formulæ describe the distribution as to height and as to income of the members of a nation. Statistics generalize and repair the defects of individual experience.

4. Statistics are specially useful for making *comparisons* of similar aggregates from time to time, or from place to place. The significance of one quantity, *e.g.* the average wage of a group of workmen, can only be appreciated by comparison with another, *e.g.* the average wage of another group in a different occupation or district, or the same group at an earlier date. The gradual *change* of the birth-, death- or marriage-rates during a series of years, shows very much more than the statements for a single year. Again, it is frequently necessary to show the *relation* of one quantity—for example, the total importation of wheat—to another, for

example, the population; or, to take another instance, the relation of the total wheat crop to the area under cultivation. The choice and exact definition of the aggregates that should be thus brought in relation to each other are by no means simple matters.

5. When observations are thus extended, many sources of inaccuracy are found to be present, and it is very frequently impossible to remove them completely. Statistical results are, therefore, very generally estimates rather than exact statements, and it is a matter of the very greatest importance to learn to what degree of accuracy various statements can be trusted, and to obtain methods of neutralizing the effects of errors and omissions of all kinds.

6. Perhaps the principal cause of incorrect use of statistics is want of attention to the definition, meaning and limitation of each estimate quoted. A total, such as the population of England and Wales, or the total value of goods imported into the United Kingdom, is generally the result of a complicated system of enumeration, in which a large body of persons have co-operated, working under printed instructions. To know what is included in the total implies not only careful reading of the title, "Total value of Foreign and Colonial Merchandise Imported," but also knowledge of the method of valuation, of the definitions of "Merchandise" and of "Imported," and of the nature of the omissions (goods brought in as personal luggage or smuggled, etc.). There is hardly any total whose full meaning is apparent simply from its description; there is always to be implied some such phrase as "so far as the items are included in the working definition and enumerated by the staff concerned." The total or average used is a total or average of many items, each of which satisfies some complex definition; this definition is not thoroughly known till the whole method of collection and tabulation is known. In many cases the necessary explanations are given in the introduction to or the footnotes of an official report; in others, where information is not forthcoming, extreme caution is necessary in

using the figures, till a careful inquiry as to their meaning can be made. In Part II below, some of the more important definitions are given.

7. It is frequently the case that the quantity as to which knowledge is desired is not capable of numerical measurements. We cannot measure health, poverty or crime; we can only measure the death-rate, and count the number of persons who receive public relief, and the number of convictions. In such cases the measurements can only be used as indications, and their relation to the more important quantity must be constantly criticized, while other indications should be obtained wherever possible to check the impressions formed. Thus the number of paupers changes with altered administration, and of criminals with modifications of law, and the death-rate differs with age, sex, locality and occupation; but in these cases we have other means of knowing and testing the changes in the quantities concerned.

8. It is very important to avoid mistaking the part for the whole. The growth of exports is often used as an indication of the general growth of trade, but the more important home trade has not been measured, and the whole may diminish while exports increase, or *vice versa*. The number of members of certain trade unions out of work is published monthly, but the percentages based on them cannot be used to measure unemployment as a whole without many qualifications. If our definitions are correct they will show the limitations in extent of our estimates. Other cautions as to common mistakes in using statistics will be found scattered through the chapters that follow.

9. Three of the principal uses of statistics are (i) to give correct views, based on facts, as to what has happened in the past; how, when and under what circumstances, population, trade, wealth, etc., have grown; and by comparison and analysis to search for the causes of changes that have taken place; (ii) to afford material for estimates for the present, *e.g.* the probable yield of a new tax, the amount of trade that will be carried by a new route, the quantity of water needed

by a town; (iii) to make possible a forecast for the near future; for this purpose we study the changes that have taken place in the recent past, by the light of the relations between phenomena that comparative statistical analysis reveals.

10. The main sources of statistical information are (i) official tables published periodically by various Government Departments, (ii) the results of special inquiries made by the Departments or by Royal Commissions or Parliamentary Committees, (iii) monthly and annual reports on special trades made by Chambers of Commerce, trade newspapers and private firms; (iv) special investigations made by private individuals as to social conditions. All of these have their limitations and present special difficulties, and together they are quite inadequate to afford sufficient information as to most of the conditions of welfare, progress and trade which form the subjects of inquiry. There is urgent need for more systematic and more complete national statistics.

11. Even if statistics were complete and perfect, their use would be definitely limited to one aspect of a problem, that is, the numerical aspect. Statistical results are essential, when judgment is to be formed on any questions that involve numbers, quantities or values, but they must always be brought into relation with the personal, political, æsthetic or other non-quantitative considerations that may be of greater importance in deciding on a course of action. Statistics only furnish a tool, necessary though imperfect, which is dangerous in the hands of those who do not know its use and deficiencies. A knowledge of methods and limitations is necessary, if only to avoid being misled by unscrupulous or unscientific arguments.

CHAPTER II

ACCURACY AND APPROXIMATION

1. PERFECT accuracy is very seldom obtained in statistics and in this respect they differ from accountancy. A statement of fact involving £ *s. d.* can be made exactly, and must be so made to afford a perfect balance, but as soon as we deal with quantities or values, where the things counted are not perfectly similar to each other or are matters of estimate, we can no longer give an exact unqualified statement. There is no means of knowing exactly the quantity of wheat grown in the United Kingdom, both because one bushel of wheat differs from another in dryness, fineness, and other respects, and because the whole bulk is not and cannot be measured, but is estimated from the acreage under wheat and the average productivity. We cannot know the population of England and Wales exactly on June 30, 1909, for it is eight years since the population was counted; no record is kept as to the numbers who have gone to or come from other parts of the United Kingdom, statistics of emigration to and immigration from the colonies and foreign countries are imperfect, and probably a small number of births are unregistered. Both these totals can, however, be estimated with considerable accuracy.

2. In such cases we should not say that the population consists of 35,751,963 persons, or that 56,531,198 bushels of wheat were produced in 1907, except as bare numerical results of a calculation; but we should aim at finding to how many figures the statements are likely to be correct.

Supposing this difficult operation performed, and that (for example) an error of 100,000 persons is possible in the

population and of 250,000 bushels in the estimated production of wheat, various methods of statement are open to us.

(*a*) The population is 35,751,963 $\pm$ a number not greater than 100,000.

(*b*) The population is 35,750,000 $\pm$ 100,000, or 3575 $\pm$ 10 (0000's omitted).

(*c*) The population is between 35,650,000 and 35,850,000.

(*d*) The population is 36×10^6, or 36,000,000, to the nearest million; or (in a table involving other similar figures) is 36 (000,000's omitted).

(*e*) The population is 35,750,000, correct to ·3%, or to 3‰.

(*f*) The population is $35^{65}_{85} \times 10^4$ (where $^{65}_{85}$ is not a fraction, but an abbreviation for "between 65 and 85").

If the error were, however, known to be not more than 2,000, we could make a shorter statement, viz. that the population is 35,750,000 "in round numbers" or "correct to 10,000"; for the maximum and minimum possible, viz. 35,753,963 and 35,749,963, are both nearer to 35,75 than to 35,74 or 35,76 (0000's omitted). This is the best method when applicable, but in the case given we cannot be sure which is the nearest 100,000, and (*d*), which is the corresponding statement, is unnecessarily rough.

Each of the above statements would be correct for some purpose; the choice depends on the nature of the table of which it is to form part. (*c*) is the clearest if we are not making a table. (*e*), or an equivalent form, is the most scientific. (*f*) has not actually come into use, but may be suggested as the most compact way in which the whole data can be stated.

3. When round numbers are used, the last digit retained must be the nearest to the estimate, not the next under.

Thus 374,563 is $374{,}56^{0}$ or 3746^{00} or 375^{000}* or 37^{0000}, not 3745^{00} and 374^{000}. In the third case, the number being

* This is merely a convenient way of writing 375,000, when it is implied that the number is correctly given only as far as the 5.

nearly midway between 374,000 and 375,000, it would be better to write 374·5 (thousands). Round numbers are employed both as abbreviations of nearly exact statements and to indicate the accuracy of estimates, the last digit given being supposed correct.

4. The arithmetic of inexact numbers needs special attention. Unnecessary work is to be avoided; only those digits should be given in the result which are supported by the premises; an indication of the possible error should be given. The following five examples show various ways in which the work can be carried out.*

Addition.—Add 47,386, 9,453, 843,782, the numbers being correct to 2 %, 5 %, and ·5 % respectively.

Then the first number is only given as between 47,386 + 948 and 47,386 − 948, and the work may be set down as follows:—

$$\begin{array}{r@{\ \pm\ }r} 47{,}386 & 948 \\ 9{,}453 & 473 \\ 843{,}782 & 4{,}219 \\ \hline 900{,}621 & 5{,}640 \end{array}$$

Answer, 900,000, correct to ·6 %, or "between 895,000 and 906,000."

If a less exact answer is sufficient, we may notice that the last entry makes the greatest contribution to the error, and write

$$\begin{array}{r} 47 \quad \text{000's omitted.} \\ 9 \\ 84^{0}_{8} \\ \hline 90 \times 10^4 \end{array}$$

Subtraction.—Subtract £85,460 from £197,000, the numbers being correct to the last digit (other than 0) given.

Then the first quantity is only given as between £85,455 and £85,465, the second as between 197,500 and 196,500.

* The concise statement given in these paragraphs will, it is hoped, be sufficient for capable arithmeticians, and a fuller treatment would be out of place; but these or similar methods are to be found in modern Arithmetics, to which the reader is referred if the ideas are not clear. In the end every one makes his own rules for abbreviation.

Work showing

the Maximum difference		the Minimum difference	
	197,500		196,500
	85,455		85,465
	£112,045		£111,035

Answer, £11½ × 10³, or £111,500, correct to ·5 %.

Multiplication.—Multiply £30 18*s.* 6*d.* by 347,100, the numbers being correct to the nearest 6*d.* and the nearest 100 respectively.

Greatest possible errors:—3*d.* in £31, or 1 in 2480; and 50 in 347,100, or 1 in 7000.

First method.

Product if there were no error.	Maximum product.
£30·925	£30·937
347,100	347,150
92775	92811
12370	12375
2165	2166
31	31
	15
£10,734,1⁰⁰	£10,739,8⁰⁰

Maximum error ± 5,700 or ·53 ‰, where ·53 ‰ stands for ·53 per mille.

Second method.—Observe that the answer can only be correct to four significant figures.

The maximum errors in the factors are ·40 ‰ and ·13 ‰. Where small percentage errors occur in factors, both being in excess or both in defect, it is easily shown by algebra or geometry that the error in the product is the sum of the errors in the factors. The product is therefore subject to an error of ·53 ‰.

```
3471
    3·0925 × 10³
────────
10,413
   312
     7
     1
────────
£10,733 × 10³
```

Answer, £10,733,⁰⁰⁰, correct to ·5 ‰ or £107¾ × 10⁴.

Third method.—A little experience will show that the

more serious error comes from the first term and is roughly ·4 ‰. The work should then be done to five figures, and the answer given as doubtful to one unit in the fourth figure.

Division.—45,340,000 tons are valued at £74,380,000. Find the value per ton, the numbers being correct to the last digit (not 0) stated.

First method.—The maximum error is obtained when the dividend is greatest and the divisor least, or *vice versa.*

Maximum possible value.	Value if there were no error.
45,335)74,385(£1·6408	45,340)74,380(£1·6405
45,335	45,340
29,050	29,040
27,201	27,204
1,849	1,836
1,813	1,814
36	22

Answer, £1 12*s.* $9\frac{3}{4}$*d.*, to nearest farthing.

Second method.—The maximum errors are 1 in 9,000 and 1 in 15,000; if cumulative they make ·11 + ·07 = ·18 ‰, that is $\frac{1}{5}$ of one farthing in £1. The quotient, worked on the supposition that there is no error, is therefore correct to the nearest farthing.

Square root.—Find the length of the side of a square field whose area is 15 a. 3 r. 29 p., correct to a square pole.

Square poles.

```
     2,549(50·488 poles = 277·68 yards.
     25
     ──
1004)4900
     4016
     ────
      884
```

The area is correct to 1 in 5000; the side can be, therefore,* obtained to 1 in 10,000, and may be stated as $277\cdot{}^{66}_{71}$ yards, or 277·7 yards.

* The *relative* error is doubled by squaring, and, conversely, halved in taking the square root. For, if x is a quantity subject to a small absolute error ex, $x(1 \pm e)$ will be the limits of the approximation to the value of x. Then $x^2(1 \pm e)^2$, which nearly equal $x^2(1 \pm 2e)$, since e^2 is small, will be the limits for the value of x^2, which is therefore subject to a relative error $2e$.

5. Multiplication, division and square root can be more rapidly performed by the use of logarithms, but there is considerable risk that part of the data will be lost, or a spurious accuracy introduced. If the data are correct to four figures, four-figure logarithms should be used, and the answer may be depended on to at least three figures, and similarly with other degrees of accuracy. Slide rules can also be used for special purposes, but their adequacy must be tested.

It is necessary to call attention to the complexity of these processes, because it is so commonly assumed that they are not worthy of attention. It is only a very competent arithmetician or experienced statistician who can see the effect of the inaccuracy of data throughout a problem. It is probable that many published statistics are less accurate than they appear, simply because the effect on the results of errors in the factors has not been considered.

It is to be observed that it is the most inaccurate of the factors or terms that governs the inaccuracy of the result.

6. Few statistical measurements are accurate to five figures, many not to more than three, and some are doubtful in the second figure. On the other hand, it is seldom that greater accuracy than 1 in 1,000 is required, and this can often be obtained.

It results that, in general, much space can be saved in tabulation and more accuracy be in reality obtained, by giving numbers only to three or four significant figures.

7. *Comparison and ratio.*—It is so much the custom to make comparisons by means of percentages, that the artificiality and, in some cases, the fallacy of the result are not perceived.

Suppose that we wish to compare two quantities, *e.g.* the aggregate values of Exports of Home Produce in 1898 (£294,014^{000}) and in 1907 (£517,977^{000}), and that we can depend on these values to four figures.

Any one of the following ratios expresses the facts—

$$2940:5180 = \mathit{1:1{\cdot}762} = {\cdot}5676:1 = 100:176{\cdot}2 = 1000:1762$$
$$= 56{\cdot}76:100 = 567{\cdot}6:1000 = 100 - 43{\cdot}24:100.$$

The ratio in italics is the simplest of these statements, if we take the value in 1898 as the standard of comparison, and that next written (·5676 : 1) if we take 1907 as the standard.

The statement most usually made would be (*a*) "The value has increased 76·2 %"; it is more exact to say "The value in 1907 was 176·2 % of that in 1898." The converse is "The value in 1898 was 56·76 % of that in 1907"; the equivalent of this is (*b*) "The value in 1898 was 43·24 % less than that in 1907." Few people would recognize that (*b*) was the converse of (*a*).*

After the phrase "per cent." the words "of x" are implied, where x is supposed to be known from the context. But the context does not always give definite information, as the following example of evidence given to a Royal Commission shows: "Wages were 15*s.* in 1870; they rose 20 % between 1860 and 1870, and 10 % more by 1875; by 1885 wages had fallen 25 %." Any of the following would satisfy the statement—

1860.	1870.	1875.	1885.
12/6	15/-	16/6	12/4½, reckoning each period by itself.
12/6	15/-	16/3	13/1½, reckoning all on the 1860 basis.
12/6	15/-	16/3	12/2¼, reckoning the last on 1875.
12/4½	15/-	16/3½	13/0½, reckoning all on the 1885 basis.

From other evidence it appears that the third of these lines was intended.

One of the greatest strikes of the end of the nineteenth century was caused by a misunderstanding of this kind.

8. It would be an improvement in common methods if the decimal point were not used in comparisons; thus the statement as to exports would read: "the values are in the ratio 1000 to 1762." It would be a greater improvement if the ratio were always given, not the increase; thus "the value

* If x and y are two numbers, y is $100 \times \frac{y-x}{x}$ = say, u per cent. greater than x, and x is $100 \times \frac{y-x}{y}$ = say, v per cent. less than y. Then the simplest relation between u and v is $100\,(u-v) = uv$.

has changed in the ratio of 1000 to 1762," not "has increased 76·2 %."

Apart from the greater definiteness of the ratio statement we gain a further advantage in preserving the measure of accuracy. If average weekly wages change from 25*s.* 9*d.* to 27*s.* 3*d.*, each quantity being given correctly to the nearest 3*d.*, the *ratio* is between 25*s.* 10½*d.* : 27*s.* 1½*d.* and 25*s.* 7½ *d.*: 27*s.* 4½*d.*, *i.e.* between 1000 : 1048 and 1000 : 1068, or may be written 1000 : 1058 ± 10, and is known to 1 %. But the *increase* is only known as between 4·8 and 6·8 %, or as 5·8 ± 1·0 %, and is doubtful to the much greater extent of 1 part in 6. This source of inaccuracy is frequently ignored.

9. There are two groups of cases in which percentages (or per thousands, etc.) can be used without indefiniteness; they can be shown sufficiently by examples—

(*a*)

Value of Imports, received by the colonies, etc., from	00000's	Per cent. of total.	or	Per mille of total.
The United Kingdom	£1434	46·4		464
British Possessions	561	18·1		181
Foreign Countries	1096	35·5		355
Total	£3091	100·0		1,000

In a long column of this sort, the percentage items, each calculated correct to the third figure, will not give in general 1,000 exactly as the total; the items should, nevertheless, be left as they are calculated.

(*b*) The second group is illustrated by the statements: "Per million males over 10 years of age in 1901 in England and Wales, 92,811 were occupied in *building and works of construction*, as compared with 34,898 per million in Ireland"; "Per thousand persons in England and Wales in 1871 and 1901, 437 and 470 respectively were between the ages 20 and 55."

Such methods of arranging numbers for comparison can hardly be distinguished from averaging, as dealt with in the next chapter.

10. The following examples illustrate common mistakes in the use of percentages—

"Of 57 persons, 35 (or 61·404 %) died." The number in the brackets is an example of spurious accuracy. In dealing with less than 100, the figure in the unit place is not established, and the decimals are absurd.

"Exports increased from £1,000 to £1,300, *i.e.* 30 %, but imports increased 500 %, the values being £20 and £120." Here are compared relative increases on values which are so different as not to be comparable; the *absolute* increase in the first case is three times that in the second. Such a statement is numerically correct, but is likely to be misquoted simply as "Exports increased 30 % and imports 500 %."

"Prices rose 20 % and then fell 20 %, returning to the former level." If the most natural meaning is given to the first clause, the three prices would be in the ratio 100 : 120 : 96 and the last price would be 4 % below the first. This kind of ambiguity and the resulting mistakes have already been discussed (p. 12).

"The total rose from about £143,000,000 to £185,473,000 an increase of 29·7 %." This should be "about 30 %."

CHAPTER III

AVERAGES

1. AVERAGES are of many kinds and have many uses. Here we deal only with the simpler averages and kindred quantities in common use, not involving mathematical analysis; and, avoiding formal definitions, we explain the methods and ideas by examples.

"1,000 cattle in the United Kingdom produce on the average 58 tons of meat per annum." We cannot say "1 cattle produces ·058 tons," for this is not true of an individual ox, cow, or calf; the use of the generic noun "cattle" itself suggests the more general statement.

An average of this kind is obtained by estimating the number of cattle and the amount of meat produced year by year over a period of years, and dividing the amount by the number.

The use of the statement is partly to abbreviate and to state in an accurate form (see last chapter) the result of a complicated investigation; partly to afford a basis by which the yield of the herds of the United Kingdom in future years can be estimated;* partly to make a standard of comparison with other countries and other dates.

2. In the census of 1901, 32,527,843 persons were enumerated in England and Wales, the area being 37,327,479 acres. There were, therefore, 0·871 persons per acre. In Worcestershire the "density" was 1·13, and in the county of London 60·62 persons per acre.

* The number of cattle is estimated by the Board of Agriculture every year; the quantity of meat is not estimated officially at all.—See *Statistical Journal*, 1909, p. 316.

This is an example of a fictitious average. To realize it, we have to make the absurd assumption that the persons are spread out over the country like butter on bread. Nevertheless, the statements in their most convenient form are of great importance for comparing the amount of land and of air space available in relation to the number of inhabitants, town by town and country district by country district.

Or, again, we may ascertain that land of certain qualities can support (say) three persons per acre on the average, and hence estimate the population that could obtain a living from a given district.

3. The population of the United Kingdom, June 30, 1907, is estimated to have been 44,099,000. The number of births registered for 1907 was 1,147,988; the birth-rate was, therefore, 26·0 (per thousand of the population per annum). Death-rates and marriage-rates are calculated in the same way. The use of these figures is for estimating the future population, for observing where the rates are abnormally high or low, so that, for example, sanitary measures may be taken with a view to reducing a high death-rate, and for studying the causes and effects of the fall in the birth- and death-rates which has been marked in recent years. These rates are averages of precisely the same nature as the yield of meat in the first example.

4. If the assessed annual value of the rateable property in a town is £900,000 and the common expenditure of the town is £300,000 per annum, a "rate" of 6*s.* 8*d.* in the £ would have to be imposed. Here the expenditure is averaged among the property-holders in proportion to the value of their property. In this case the average (expenditure ÷ assessed value) must be obtained first, and then the sum payable in respect of each property is calculated.

In 1910 the national expenditure of the United Kingdom was about £160,000,000, the population about 45,000,000; the aggregate of personal incomes was estimated as £2,000,000,000, but cannot be known within 10%. On these figures the necessary tax per head would be £3 11*s.* if all the money

were collected directly in equal amounts, person by person, and would be 1*s.* 5*d.* to 1*s.* 9*d.* in the £ if it were collected directly in proportion to income. By such averages an individual can estimate whether he is paying his due share of the national burden.*

The averages so far used are typical examples of arithmetical averages. An "arithmetical average" is usually defined as the quotient obtained by dividing the sum of several items by the number of items; this may be extended to include the quotient obtained by dividing a total by the number of persons or things connected with it.

5. If 25 lbs. of tea at 2*s.* are mixed with 50 lbs. at 1*s.* 6*d.*, the cost of the mixture is 1*s.* 8*d.* per lb. Conversely, if the prices of the constituents and the cost per lb. of the mixture were given, a simple arithmetic process shows that the proportions by weight of the constituents were as 1 to 2.

[Weight of dearer : weight of cheaper = Average − price of cheaper : price of dearer − average.]

If 100 unskilled workmen at 25*s.* and 50 skilled at 37*s.* are employed, the average wage per workman is—

$$\frac{100 \times 25s. + 50 \times 37s.}{150} = 29s.$$

The last illustration is an example of a "weighted average," the numbers 100 and 50 being the weights in this case; the same process can, of course, be used for combining several groups.

A "weighted average" is obtained as follows:—Each of a series of quantities is multiplied by the number of persons or things connected with it, these multipliers being called "weights"; the sum of these products is taken as numerator, the sum of the weights as denominator; the fraction is the weighted average.

Examples and theory † show that slight errors in the

* Actually the problem is very difficult, since a great part is obtained in indirect taxation.

† *Elements of Statistics*, pp. 205–212 and 304.

"weights" have little effect on the average, if a fairly large number of terms are involved, none of them preponderant; and it frequently happens that the weights must be estimated, while the wages (or the other numbers concerned) are known accurately. Further, it is only necessary to know the *ratio* of the weights to each other, as a little consideration will show. If w_1, w_2, w_3 are weights, and n_1, n_2, n_3 numbers, the weighted average is $\frac{w_1n_1 + w_2n_2 + w_3n_3}{w_1 + w_2 + w_3}$. If the weights are changed, by multiplying each by k, to kw_1, kw_2, kw_3, the weighted average is $\frac{kw_1n_1 + kw_2n_2 + kw_3n_3}{kw_1 + kw_2 + kw_3}$, which clearly equals the former fraction.

One practical result of this principle is that the weights may be expressed in round numbers.

EXAMPLE.

Populations.	Numbers of Agricultural Labourers. (1)	Weekly Wage.	Weights. (2)	Weights. (3)
16,060	4,123	13*s.* 6*d.*	41^00	8
18,300	4,527	14*s.* 0*d.*	45	9
20,500	4,802	16*s.* 0*d.*	48	10
22,600	5,432	15*s.* 6*d.*	54	11

If weights (1) are taken, the average wage is found to be 14*s.* 10·0*d.*; if the round numbers (2) are used, the average is 14*s.* 10·1*d.* If it is observed that the numbers of labourers are nearly proportional to the populations, and if the weights (3), which are also nearly proportional to the populations, are used, the average is 14*s.* 10·3*d.*

The effect of taking approximate numbers for weights should always be carefully tested before the result is accepted.

6. In calculating averages of this kind, the work can often be greatly abbreviated without affecting its accuracy by either of the methods used in the following example. The proofs are left to the student.

CALCULATION OF THE AVERAGE WAGE OF THE GROUP WHOSE WAGES ARE SHOWN IN COLUMNS 1 AND 2.

1. Numbers.	2. Wages.	3. Wages 8s.	4. Product of Columns 1 and 3.	5. Wages 18s.	6. Product of Columns 1 and 5.	
27	8s.	+ 0s.	0	− 10s.	− 270	
23	10s.	2	46	− 8s.	184	
28	11s.	3	84	− 7s.	196	
41	12s.	4	164	− 6s.	246	
45	13s.	5	225	− 5s.	225	
49	14s.	6	294	− 4s.	196	
58	15s.	7	406	− 3s.	174	
61	16s.	8	488	− 2s.	122	
65	17s.	9	585	− 1s.	65	
65	18s.	10	650	0	—	
65	19s.	11	715	+ 1s.	—	+ 65
65	20s.	12	780	+ 2s.	—	130
62	21s.	13	806	+ 3s.	—	186
51	22s.	14	714	+ 4s.	—	204
48	23s.	15	720	+ 5s.	—	240
40	24s.	16	640	+ 6s.	—	240
33	25s.	17	561	+ 7s.	—	231
21	26s.	18	378	+ 8s.	—	168
16	27s.	19	304	+ 9s.	—	144
26	30s.*	22	572	+ 12s.	—	312
889			9132		− 1678	+ 1920 = 242

Using Column 4—

$$\text{the average wage is } 8s. + \frac{9132}{889}s. = 18s.\ 3\tfrac{1}{2}d.$$

Using Column 6—

$$\text{the average wage is } 18s. + \frac{2\ 2}{889}s. = 18s.\ 3\tfrac{1}{2}d.$$

Columns 3 and 5 are equivalent to Column 2. In 3, 8*s.* is taken simply because it is the minimum entry. In 5, inspection of the figures shows that the average is likely to be between 16*s.* and 20*s.*; 18*s.* was chosen as the starting point, as it appeared (without working) to be just below the average; the nearer the point chosen to the average, the less the numerical work required.

* Actually, "28*s.* or more."

. The following table is a condensation of the one just given, and is suitable for rough, but fairly accurate, work.

Wages.	Numbers.	(a) Numbers.	Wages.	(b) In 5s. units.	Product of (a) and (b).
Below 10s.	27	3	say 7s.	7s. + 0	0
10s. and below 15s.	186	19	12s.	1	19
15s. " " 20s.	314	31	17s.	2	62
20s. " " 25s.	266	27	22s.	3	81
25s. " " 30s.	70	7	27s.	4	28
30s.	26	3	30s.	4·6	14
		90			204

$$\text{Average, } 7s. + \frac{204}{90} \text{ of } 5s. = 18s.\ 4d.$$

Here 12*s.*, 17*s.*, etc., are taken as the middle wages of the groups 10*s.* to 14*s.*, 15*s.* to 19*s.*, etc. If the wages were not in exact shillings, but were originally given as "23 persons earning 10*s.* and less than 11*s.*" etc., then 12*s.* 6*d.*, 17*s.* 6*d.*, etc., should be taken for the middle wage of the groups.

7. In distinction to the "arithmetical averages" described in paragraphs 1–5, which are mainly of use in facilitating further arithmetical processes, that in paragraph 6 may be called a descriptive average, for it can be used as an abbreviated way of describing the "group" of wages in the table.

The following sentences contain nine descriptive * averages. From the Board of Trade inquiry as to rents, prices and wages in the towns of the United Kingdom,† we learn that the average family weekly income was 36*s.* 10*d.*, the average number of children living at home was 3·6, the total expenditure on food was 22*s.* 6*d.*, of which 4*s.* 5½*d.* and 3*s.* 7*d.* were used for the purchase of 6·5 lbs. of meat and 32·0 lbs. of bread and flour respectively. The average rent for a five-roomed house outside London was about 6*s.*

* This word is not in general use as a technical term, but may be suggested as useful in classifying averages.

† Cd. 3864 of 1908.

Such averages are usually calculated by adding the total wages (expenditures, quantities, etc.) and dividing by the number of instances; that is, they are arithmetical averages, or (where the method of paragraph 5 has been used) weighted averages.

An alternative method of description would be to find out, *e.g.* the size of house which was *most commonly* used by the working-class; thus, if we know that 15, 25, 50, and 10 % of the families inhabited 3-, 4-, 5- and 6-roomed houses respectively, the 5-roomed house would be most usual or "predominant." We might further determine that (say) 6*s.* 6*d.* was the "predominant" rent. Our whole description might then be given in terms of "predominant" wages, rents, etc. As a sure description this is more vivid than the former; we should be describing the family of which, in fact, there were most instances, instead of an artificial family with 3·6 children. Such predominant rates are in statistics regarded as averages, and are technically called "modes" (fashionable, common).

The "mode" may be defined as that value of the graded quantity (wages, years, etc.) at which the instances are most numerous. Very generally in the statistics with which this book deals the apparent position of the mode depends on the accident of grading, and the mode cannot be exactly determined even by mathematical analysis.

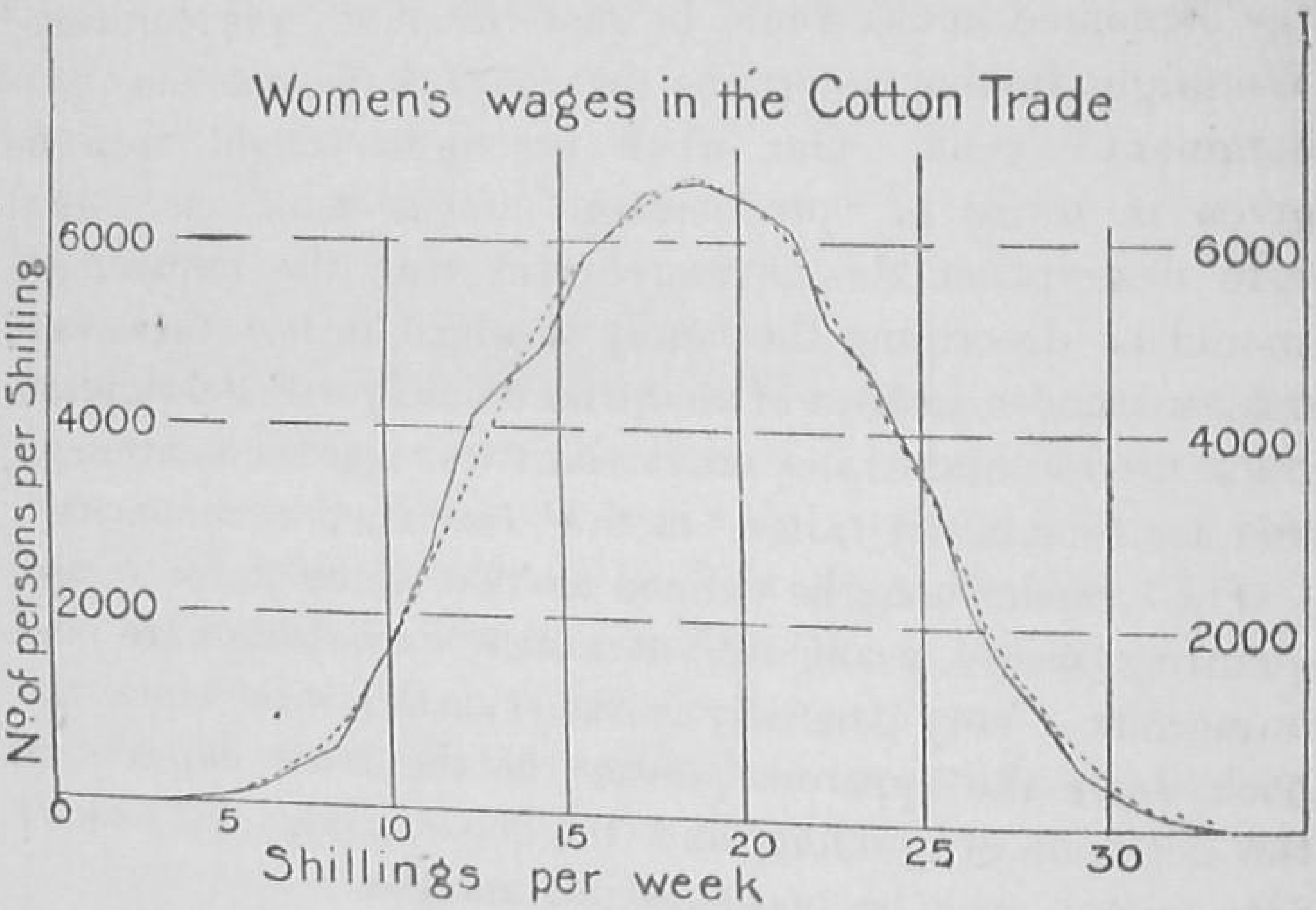
Another objection to its general use is that it is not obtained by a simple arithmetic process, and cannot be used, like arithmetical averages, for obtaining totals: if the arithmetical average of 3,000 men's wages is 30*s.*, the total wage is £4,500, but if we are told only that the "mode" is 30*s.* we cannot calculate the total.

The "mode" is more useful in anthropometrical and biological statistics, where there is a definite type, from which the measurements of the individuals of a group show deviations; in such cases the position of the mode affords precisely the measurement that defines the type.

Sometimes the word *average* is restricted to merely arithmetical measurements, while the word *mean* is used when a

group is described; if this distinction were made, "modes" and "medians" (see p. 24) would be means. But there is no general agreement on this point, and French and German writers do not make a corresponding distinction; we therefore regard the words as synonymous.

8. The group of wages given in paragraph 6 is a slightly modified statement of the weekly wages of women in the cotton industry. More complete figures are represented on the adjoining diagram. Such a diagram showing vertically



the relative numbers corresponding to the wages (ages, size or other measurements) marked on the horizontal scale is known as a "frequency curve," and a great part of more advanced statistics deals with such curves, which show the frequency of the occurrence of examples at various measurements.* Here we will only observe that the complete description of a group can only be given by such a curve or by an elaborate table, and that averages or means are only a shorthand or abbreviated way of describing some important characteristics of the group. The arithmetical average, which

* The dotted line in the diagram shows the effect of smoothing off the angles of the broken line; the latter represents the data as given.

shows on the horizontal scale the position of the centre of gravity of the area contained by the curve, and the "mode," which shows on the horizontal scale the position of the highest point, have already been discussed; in this case we certainly cannot obtain the latter correct to 1*d.*, as we can the former.

In paragraph 6 we assumed for simplicity that the wages were exactly at 10*s.*, at 11*s.*, etc. More accurately we now read the column as "10*s.* and under 11*s.*," "11*s.* and under 12*s.*," etc. Women's wages in the cotton trade are to a large extent piece-rates, calculated out to ½*d.*, and do not tend to arrive at exact shillings. The arithmetical average is in fact (as given in the Report, Cd. 4545, p. 28) 18*s.* 8*d.*, which we should have obtained if we had assumed that the average for such a group at "11*s.* and under 12*s.*" was 11*s.* 4½*d* and so on; actually some of the women are paid exact shillings, but many are paid by the piece and their earnings amount to any odd money; in the illustrative work we took it as 11*s.*, etc. No general rule can be given for such approximation; each case must be understood and judged on its merits.

9. Now make a new table from these figures as follows—

Total (or cumulative) number.			Total (or cumulative) number.		
Earning under	11*s.*	50	Earning under	21*s.*	592
	12*s.*	78		22*s.*	654
	13*s.*	119		23*s.*	705
	14*s.*	164		24*s.*	753
	15*s.*	213		25*s.*	793
	16*s.*	271		26*s.*	826
	17*s.*	332		27*s.*	847
	18*s.*	397		28*s.*	863
	19*s.*	462		All	889
	20*s.*	527			

Consider the values of *a*, *b*, *c* in the following statements; '*Half the wage-earners received a/- or less, one quarter received b/- or less, one quarter received c/- or more.*"

To determine *a* we want the position of the 445th worker (in order of wages from the beginning). The 397th worker just failed to reach 18*s.*; but 65 earned from 18*s.* to 19*s.*

and we need the 48th up this group. Making the not unreasonable assumption * that 65 were distributed uniformly 1*d.* by 1*d.* from 18*s.* to 19*s.*, we find that the 48th was at 18*s.* 9*d.*

The work may be shown as follows—

$$a/- = 18s. + \frac{445 - 397}{65} \text{ of } 1s. = 18s.\ 9d.,$$

$$\text{similarly, } b/- = 15s. + \frac{222 - 213}{271 - 213} \text{ of } 1s. = 15s.\ 2d.,$$

$$\text{similarly, } c/- = 22s. + \frac{667 - 654}{705 - 654} \text{ of } 1s. = 22s.\ 3d.$$

a/- is called the "median," *b*/- and *c*/- are the lower and upper "quartiles" for this wage group.

The median and quartiles of a group may be thus defined: If the members of the group are ranked in order according to the measurement (wages, ages, height, etc.) under consideration, then the measurements of the members most nearly one quarter, one half and three quarters respectively along the rank are the "lower quartile," the "median" and the "upper quartile."

Such quantities obviously afford a very simple and definite description of a group. In fact, this method is the most helpful of the statistical abbreviations, and it is rapidly coming into common use.

The main objection to the median, as to the "mode," is that it does not lend itself to further numerical work. The following statement is true of the arithmetical average, but not necessarily of the median or mode—

If a_1, a_2 are the average wages of two groups of n_1, n_2 persons, then $\frac{n_1a_1 + n_2a_2}{n_1 + n_2}$ is the average for the combined group.

10. *Fallacies*—i. The average rate of a journey where alternate miles are done at 8 and 12 miles per hour, is not 10 miles

* Actually there is some concentration at 18*s.*; with full information this should be taken into account.

per hour, but 9·6 miles per hour, for two successive miles occupy $12\frac{1}{2}$ minutes.* The average rate of increase when three successive annual increments are 20 %, 30 % and 40 %, is not 30 %, but

$$\sqrt[3]{(1{\cdot}20 \times 1{\cdot}30 \times 1{\cdot}40) \times 100}\,\dagger - 100 = 29{\cdot}75\ \%.$$

ii. The average rate of interest of three sums of money bearing 3, 4 and 5 % respectively, is not necessarily $\frac{1}{3}(3 + 4 + 5) = 4\ \%$; *e. g.* if the sums are £1,000, £3,000 and £8,000 respectively, the interests are £30, £120 and £400, and the average rate is $4\frac{7}{12}$ %. "Weights" cannot be neglected without examination, nor unless certain special conditions are satisfied.

iii. If three groups of men have their wages raised each 20 %, the average is not necessarily also raised 20 % unless the relative numbers in the groups are unchanged. This is shown by the following example in which the average actually falls—

	At first Date.		At second Date (Wages increased 20%, but relative Nos. changed).	
	Numbers.	Wages.	Numbers.	Wages
Group 1.	100	20*s.*	400	24*s.*
Group 2.	200	25*s.*	200	30*s.*
Group 3.	400	40*s.*	100	48*s.*
Total	700	Average $32\frac{6}{7}$*s.*	Total 700	Average $29\frac{1}{7}$*s.*

Neglect of a change of weights always distorts and sometimes reverses the results.

Again—

	Population A. Number.	Population A. Death-rate.	Population B. Number.	Population B. Death-rate.
	44,000	16·4	44,000	16·2
Components:				
Under 5 years	4,000	25·5	1,000	26·0
Over 5 years .	40,000	15·5	43,000	16·0

Here the death-rate of Population A as a whole is higher than that of B, though the rates of the two parts shown are each lower; for A contains a larger proportion of young children, for whom the rate is high. ‡

* 9·6 is the harmonic mean between 8 and 12.

† The geometric mean.

‡ In connection with this example see the method of correcting the death-rate, p. 106, below.

In using arithmetic averages for the comparison of two groups, it is necessary to analyse the groups, and find if they are sufficiently homogeneous (of the same kind) in themselves to allow a reasonable comparison.

iv. *False accuracy.*—The average wage of two groups, the first of 100 men whose average is stated to be between 25*s.* and 26*s.*, the second of 200 men whose wage is between 30*s.* and 31*s.*, is not known to be

$$\frac{100 \times 25/6 + 200 \times 30/6}{300} = 28s.\ 10d.,$$

but is only known as between 28*s.* 4*d.* and 29*s.* 4*d.* Where there are many items, the average is more accurate than its constituents, but not necessarily when there are only two or three.

11. In an average the constituents of the numerator should be similar in kind to each other, and so should the constituents of the denominator. Also the various parts of the denominator should bear similar relations to the parts of the numerator. It is thus correct to speak of the death-rate of a population of healthy male adults, for they are subject to similar risks; it is correct to speak of the average wage of men in a trade. As we extend our view to include the whole population or a large group of trades, more and more caution is needed in the use of the average, though there are problems in which these wide averages are useful. It is doubtful whether any use can be made of the average frequently stated: "Total imports and exports divided by the population," as measuring the amount of foreign trade; for imports and exports are of different, even opposite, kinds for most practical purposes, and do not concern equally all the members of a population. Similarly "the average income per head of the population" can only be used for arithmetical purposes, not (except in a few cases) for comparison of one population with another.

CHAPTER IV

THE ACCURACY OF AVERAGING AND OTHER ARITHMETICAL PROCESSES

POPULATION OF THE COUNTY OF LONDON

	(1) Enumerated	(2)	(3) Nearest 1000	(4)	(5) Next 1000 under	(6)
	1851	1901	1851	1901	1851	1901
			000's		000's	
City of London .	127,869	26,923	128	27	127	26
Battersea . . .	10,560	168,907	11	169	10	168
Bermondsey . .	85,308	130,760	85	131	85	130
Bethnal Green .	90,193	129,680	90	130	90	129
Camberwell . .	54,667	259,339	55	259	54	259
Chelsea	54,078	73,842	54	74	54	73
Deptford . . .	24,899	110,398	25	110	24	110
Finsbury . . .	125,418	101,463	125	101	125	101
Fulham	11,886	137,289	12	137	11	137
Greenwich . . .	47,377	95,770	47	96	47	95
Hackney . . .	53,589	219,272	54	219	53	219
Hammersmith . .	17,760	112,239	18	112	17	112
Hampstead . . .	11,986	81,942	12	82	11	81
Holborn	95,676	59,405	96	59	95	59
Islington . . .	95,329	334,991	95	335	95	334
Kensington . . .	44,403	176,628	44	177	44	176
Lambeth	139,325	301,895	139	302	139	301
Lewisham . . .	18,616	127,495	19	127	18	127
Paddington . . .	48,415	143,976	48	144	48	143
Poplar	47,162	168,822	47	169	47	168
St. Marylebone .	157,696	133,301	158	133	157	133
St. Pancras . . .	166,956	235,317	167	235	166	235
Shoreditch . . .	109,257	118,637	109	119	109	118
Southwark . . .	152,371	206,180	152	206	152	206
Stepney	238,910	298,600	239	299	238	298
Stoke Newington .	6,076	51,247	6	51	6	51
Wandsworth . .	40,204	232,034	40	232	40	232
Westminster . .	244,178	183,011	244	183	244	183
Woolwich . . .	43,177	117,178	43	117	43	117
Total of the 29 districts	2,363,341	4,536,541	2,362	4,535	2,349	4,521
Averages. . . .	81,494	156,432	$81,45^{0}$	$156,38^{0}$	$81,00^{0}$	$155,9^{00}$
Ratios 1851 to 1901	1000 : 1920		1000 : 1920		1000 : 1925	

1. The word error is used in statistics, not as meaning a mistake, but as denoting the difference between an estimate and the generally unknown exact measurement. We must distinguish between two methods of measuring error. In the adjoining table, the population of the county of London is shown as 4,536,541 in column (2), and estimated as 4,535,000 in column (4); the difference, 1,541, is called the *absolute* error; the ratio of 1,541 to 4,535,000, *i. e.* ·00034, is called the *relative* error. The *relative* error may also be expressed as a *percentage* error, in this case ·034 %. No simple rule can be assigned as to when absolute and when relative errors are the more important.

In the table, columns (1) and (2) give the populations of the city of London and the Metropolitan Boroughs in 1851 and 1901. Columns (3) and (4) give the same numbers to the *nearest* thousand; columns (5) and (6) give the same numbers *omitting* the last three figures in each case.

Example of *absolute* errors.—The average of the successive numbers 0 to 999 is 499·5. In numbers stated as in columns (5) and (6) we are equally likely to have omitted any number from 0 to 999, and are liable to an absolute error which cannot be greater than 29 × 999 or less than 0, and whose most probable value is 29 × 499·5 = 14,500 (nearly). The errors are actually 14,341 and 15,541, as may be seen from columns (1) and (2).

Example of *relative* errors.—The relative errors in column (6) are the ratio of numbers varying from 0 to 999, with average value very nearly 500, to the numbers in the column (26,000, 168,000, etc.). The smaller the population the greater the probable relative error. In the first line it is nearly $\frac{1}{26}$, while for Stepney it cannot be so great as $\frac{1}{298}$. There is no simple relation between the relative errors in the items and in the total, except that the latter is between the greatest and least of the former.

2. It is clear that columns (5) and (6) under-estimate all the items and the total, while columns (3) and (4) are equally likely to be in excess and defect. Such errors as the latter

are called *fortuitous* or *unbiassed* errors, while the former (which all tend in the same direction) are *biassed.* The simple total of the absolute biassed errors in column (6) is the absolute error in the total. The case is very different for the unbiassed errors of columns (3) and (4). It is just as likely that they will be subtractive as additive ; actually 14 of the numbers in column (4) and 13 in column (3) are in excess, while 15 and 16 respectively are in defect. It is obvious that these errors possess a strong tendency to neutralize each other, but it is not obvious to what extent this neutralization will take place.

[Paragraphs 3 and 5 can be omitted without losing the sequence of the other paragraphs.]

3. The following rules must be accepted at present without proof, but they certainly appear plausible, and can be confirmed by experiment.

In the case of unbiassed errors—

(*a*) The *absolute* error in the total *increases* with the number of items, when each is subject to the same unbiassed absolute error.

(*b*) The best estimate for the absolute error in the total is the average absolute error to which the items are liable, multiplied by the square root of the number of items.

(*c*) The *relative* error in the total *diminishes* with the number of items.

(*d*) The best estimate for this relative error is the average absolute error of the items multiplied by the square root of the number of items and divided by the total.

(*e*) It is better to write (*d*) :—The best estimate for the relative error of the total is the average absolute error of the items divided by the average of the items, and also by the square root of the number of items.

Examples of (*a*) *and* (*c*).—If the first 4 lines only of column (4) are added, the absolute and relative errors are

respectively 730 and $\frac{1}{625}$, while those for the 29 lines are 1,541 and $\frac{1}{2944}$.

Examples of (b).—The average absolute error to which the items in col. 4 are liable is very nearly 250, all numbers from 0 to 500 being equally probable in the table above. The best estimate for the error of the sum (if we know nothing further about it) is $250\sqrt{29} = 1{,}346$, and the sum may be written $4{,}535{,}000 \pm 1{,}346$.* Actually column (2) shows that the true value is just outside this margin. The total for column (1) is just inside the similar margin ($2{,}362{,}000 \pm 1{,}346$) obtained from column (3). We must not expect in general to be just at the margin.

Examples of (d) and (e).—The average of the items in column (4) is 156,400; their average absolute error is 250; their number 29. The relative error in the total is then estimated

from (*d*) as $\frac{250\sqrt{29}}{4{,}535{,}000}$, and from (*e*) as $\frac{250}{156{,}400\sqrt{29}}$, since 156,400 is the average item. Each of these = ·0003. The relative error found by comparing columns (2) and (4) is ·00034.

Similarly the computed relative error in column (3) is ·0006, and that found from columns (1) and (3) is ·0005.

Of course there is no means of determining what the error actually is when we only know the estimates. These rules only afford a means of estimating the errors to which we are liable.

4. The averages given in the last line but one of the table are of no importance except for illustrating the principles of this chapter.

It is evident that the absolute error of the average equals the absolute error of the total divided by the number of items, in this case 29.

It should also be evident that the relative error of he average is exactly equal to the relative error of the total.

* More exactly this means, "it is as likely as not that the total is within these limits, and very unlikely that it is as much as (say) six times as far from the estimate (4,535,000) as these limits are. The most probable value is 4,535,000, in the absence of information."

Biassed errors then remain in the average unaltered. The absolute error of the average will be very near the average absolute error of the items. Thus for both columns (5) and (6) the average errors may be expected to be (see paragraph 1) 500. We should therefore estimate the averages as $81,000 + 500 = 81,500$ for 1851, and $155,900 + 500 = 156,400$ for 1901, and these estimates differ very little from those shown in columns (1) and (2).

Unbiassed errors tend to disappear in the average just as they tend to disappear in the total. In fact, the absolute errors in the averages of columns (3) and (4) are only 44 and 52, and the relative errors ·0005 and ·00034 respectively.

5. The rules of paragraph 3 become for averages—

In the case of unbiassed errors—

(*b*) The best estimate for the absolute error of an average is the average absolute error of the items divided by the square root of their number; viz. $\frac{250}{\sqrt{29}} = 46$.

(*e*) The best estimate for the relative error of an average is the average absolute error of the items divided by the average of the items and also by the square root of their number, viz. $\frac{250}{156,400\sqrt{29}} = \cdot 0003$ as before.

6. As a further illustration of biassed errors it may be noted that to obtain round numbers in a long addition of n items, we may carry $\cdot 45n$ from the unit column to the tens, instead of doing the addition, since 4·5 is the average of the digits 0 to 9. From the hundreds column we may carry $\cdot 5n$, since 50 is very nearly the average of the numbers 0 to 99. Similarly in adding money we may add $5\frac{1}{2}d. \times n$ for the pence, and $9s.\ 6d. \times n$ for the shillings, if the items end in pence and shillings respectively. If both pence and shillings are given we add $10s. \times n$ to the £.

Thus in column (1) by this rule the numbers to carry would be $4{\cdot}5 \times 29 = 13$, $\cdot 5 \times 29 = 14$ or 15. Actually the numbers carried are 16, 16, 14, 15 in order.

7. *Comparison of similar totals or averages.*—Here we only deal with relative error. The actual ratio of growth shown in columns (1) and (2) is 1 : 1·920. That shown in columns (5) and (6) is 1 : 1·925. The relative error is $\frac{5}{1925}$ = ·0026. The relative error is identical for averages and for totals.

The relative error of the ratio is very nearly equal to the difference between the relative errors of the two terms. If the errors are both positive or both negative, as is the case with biassed errors (unless there is a change of bias), the error in the ratio is less than that of the terms. Thus the relative errors for the totals of columns (5) and (6) are ·0061 and ·0032 respectively, both in defect; the difference is ·0029 very nearly the same as ·0026 just given.

There is no reason to expect that the small errors resulting from the addition of unbiassed errors will be both in excess or both in defect, though it happens to be the case in columns (3) and (4). In general we may expect the error resulting from unbiassed errors to be slightly greater in the ratio than in the terms.

The general result is that unbiassed errors tend to disappear in the averages and not to reappear in the ratio, while biassed errors tend to disappear in the ratio. The comparison of averages well constructed on similar principles generally has great accuracy, greater than that of the original items or totals. It has already been pointed out that the process of "weighting" also leads to accuracy. In fact, the ratio of weighted averages can under certain conditions which are often realized be obtained with a surprising accuracy. It can generally be determined by experimenting with the numbers whether these conditions are present.*

8. In dealing with a group, as in the last chapter, it is to be noticed that there may be a good deal of uncertainty about the extreme parts of the group, and yet the averages may be well determined. Thus the "mode" is not influenced

* For an example, see *Statistical Journal*, 1906, pp. 164 *seq.*

at all by anything except the central portion. The median is known completely for the table on p. 19, if the numbers (say) above 25*s.* and below 15*s.* are given, but not the exact wages in these marginal groups, and if numbers and wages are given in the central region; even if the top group, 26 at 28*s.* or more, were dropped out entirely, the median would only be lowered from 18*s.* 9*d.* to 18*s.* 7*d.* The arithmetical average is more easily affected by the position and magnitude of the extremes, especially the upper extreme; if of the 26 at 28*s.* or more (whose average in fact, is near 30*s.*), 6 were at 35*s.*, and 10 each at 40*s.* and at 50*s.*, the average would be raised from 18*s.* 8*d.* to 19*s.* 1*d.*; in such cases, general knowledge of the structure of the group will often make possible the assignment of narrow limits within which the average must lie.

9. When only two or three or a few terms are present, the rules given as to approximate work and round numbers in Chapter II apply. The greatest absolute error in the terms of an addition or subtraction, or in a factor of a product, dominates the error in the result. Many terms (say 20 or more) are necessary before the fortuitous errors can be confidently expected to neutralize each other. Of course, paragraph 7 above applies if the two terms form a ratio. The general practical rule in all cases involving few terms is to work through the problem, assuming every error is as great as possible under the conditions of the question, the sign of the errors being so chosen that they all work towards increasing the error in the result. Then give the answer in one of the forms of p. 7; if sufficient accuracy for practical purposes can be attained by giving the nearest round number which is certain, the statement "correct to the last digit given" is the best.

10. That a small absolute error in an item may have a great effect on the result may be illustrated by the following examples—

(*a*) Cost of workmen's budget.

	Prices. 1st date.	Prices. 2nd date.
Meat, 8lbs. . . .	$8\frac{1}{2}d.$	$9\frac{1}{2}d.$
Bread, 20lbs. . . .	$2\frac{1}{2}d.$	$2\frac{1}{4}d.$
Total	9*s*. 10*d*.	10*s*. 1*d*.

Suppose that the price of bread had been obtained as an average 2·38*d*., and had then been written to the nearest farthing, viz. as $2\frac{1}{2}d.$

Now suppose that a slight mistake had been made in the working of the average for this price, everything else being correct, and in fact it should have been 2·37*d*. Given to the nearest farthing this is $2\frac{1}{4}d.$ The first budget would then have amounted to 9*s*. 5*d*., and the increase would have appeared as + 8 %. In this case a relative error of not more than 1 in 200 results in a relative error of 5 in 3.

A careful writer would have said in this case that there was no certainty of any change in the total.

(*b*) Of 695,720 members of Trade Unions, 7·4 % were unemployed at the end of September 1909. Seven groups of trades account for 579,899 members, of whom 8·5 % were unemployed. Can the number be deduced for the remaining group?

At first sight we might proceed as follows—

	Members.		Unemployed.
7·4 % of	695,720	=	51,483
8·5 % „	579,899	=	49,291
Residue	115,821		2,192

But the 7·4 and 8·5 are more exactly from the original figures 7·42 and 8·455, the total number unemployed was 51,749, and that for the seven groups 49,028. The residual number was therefore 2,721, which exceeds the estimate (2,192) by 25 %.

CHAPTER V

USE OF DIAGRAMS

1. DIAGRAMS do not add anything to the meaning of statistics, but when drawn and studied intelligently they bring to view the salient characteristics of groups and series they show the various parts in relation to each other and to the whole, bring to light the unity that underlies the scattered figures, and suggest in what directions investigation is needed. Merely pictorial diagrams are not only unlikely to be of much use, but in advertisements and political propaganda are often deliberately misleading, though literally correct. In the author's opinion the graphic method should rarely be used except (i) to show the relations of one part of a *group* to another (the word used in the sense of p. 1, where the various members differ in respect of one measurable characteristic), (ii) to exhibit a *series* of similar estimates date by date, (iii) to compare two or more groups, (iv) to compare two or more series, (v) to exhibit three relations which can be geometrically united.

Diagrams which simply show relative magnitudes—*e. g.* the populations of three countries at one date, or two isolated figures, such as the sale of some commodity at two dates, where the horizontal scale shows no graduated quantity (time, age, wage, height, etc.)—are of no assistance for the comprehension of the numbers.

Nevertheless, a skilful writer can often devise statistical diagrams of other kinds which help the visualization of a complex argument, and the aid received from diagrams varies greatly from person to person, so that it would be rash to lay down too rigid rules.

2. The four pages of diagrams given illustrate the main principles of graphic statistics and afford examples of most of the methods that are to be recommended. The first shows the two ways of representing a *group;* one is chosen that presents some difficulties, in order to show the elasticity of the method.

AGES OF MEN AND BOYS EMPLOYED IN COAL MINES, ENGLAND AND WALES, 1901.

Age.	Number.	Per Mille.		Cumulative
10-14 years	2,761	7	Under 14 years	7
14-15	3,992	10	15	17
15-20	36,469	89	20	106
20-25	67,349	164	25	270
25-35	131,818	322	35	592
35-45	86,735	212	45	804
45-55	53,305	130	55	934
55-65	22,073	54	65	988
65-75	4,645	11	75	999
75 and over	382	1	Total	1,000
	409,529 =	1,000		

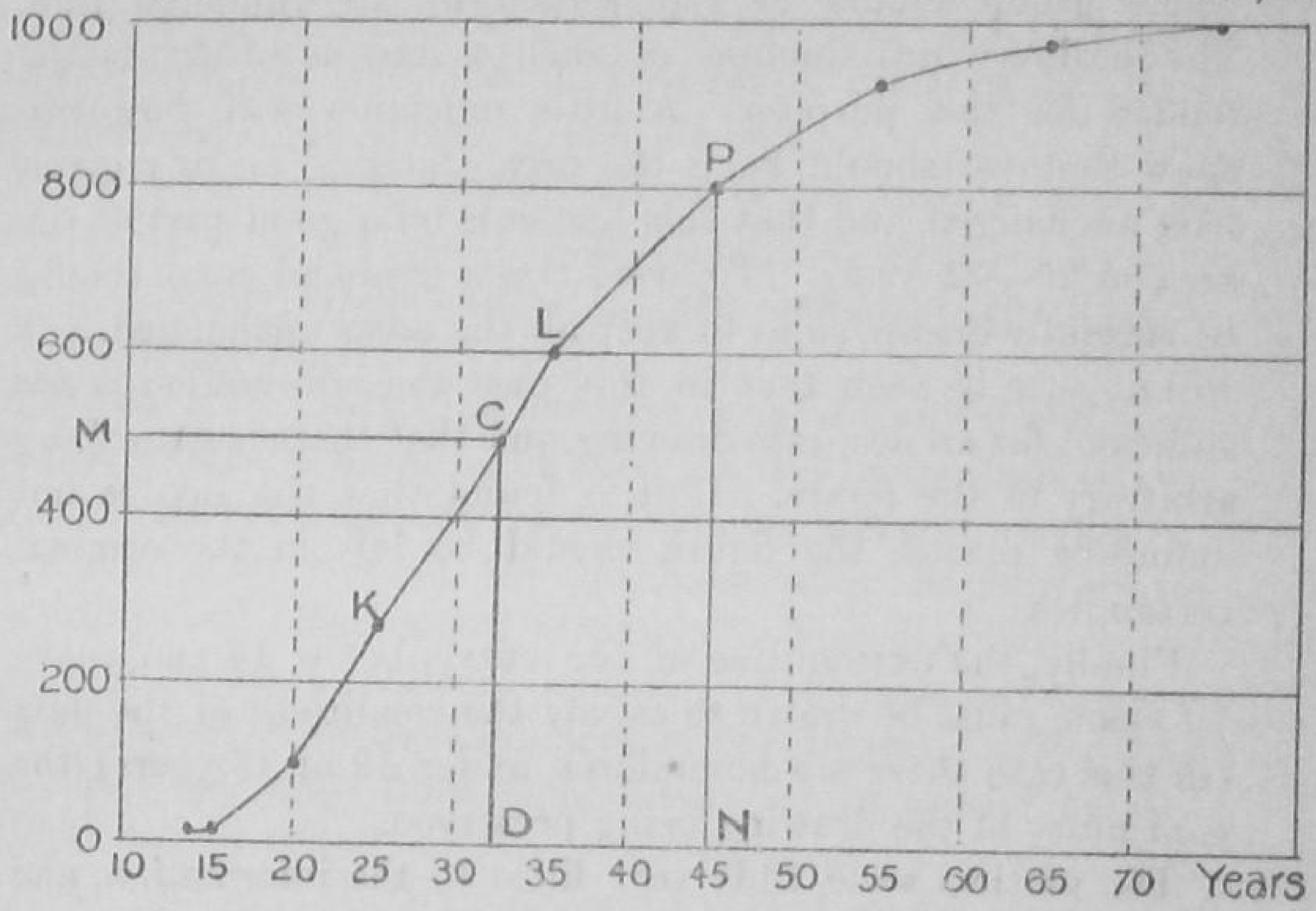
The ordinary way of showing such a group graphically is that of B on the page opposite. The years are marked off on a horizontal scale. The numbers in the six equal age periods (15—25 years, 25—35 years, etc.) are represented by rectangles proportional to these numbers on any convenient vertical scale. It is customary, but inaccurate, to join the middle points (*a*, *b*, *c*, *d*, *e*, *f*) on the tops of these rectangles by straight lines, as in the figure. If there are many narrow rectangles, as in the diagram (p. 22) above, the inaccuracy is slight, and may be ignored.

In diagram B $\frac{1}{10}$th inch square represents 4 per 1,000 of the persons throughout. It is not difficult to see that if we represent the numbers at 15—20 years and 20—25 years separately, we must keep the same areal relation by doubling the vertical scale. Similarly, if the number at 14—15 years were shown, it would be represented by a vertical scale increased tenfold. This method will become clear as soon as an attempt is made to draw the diagram from the numbers.

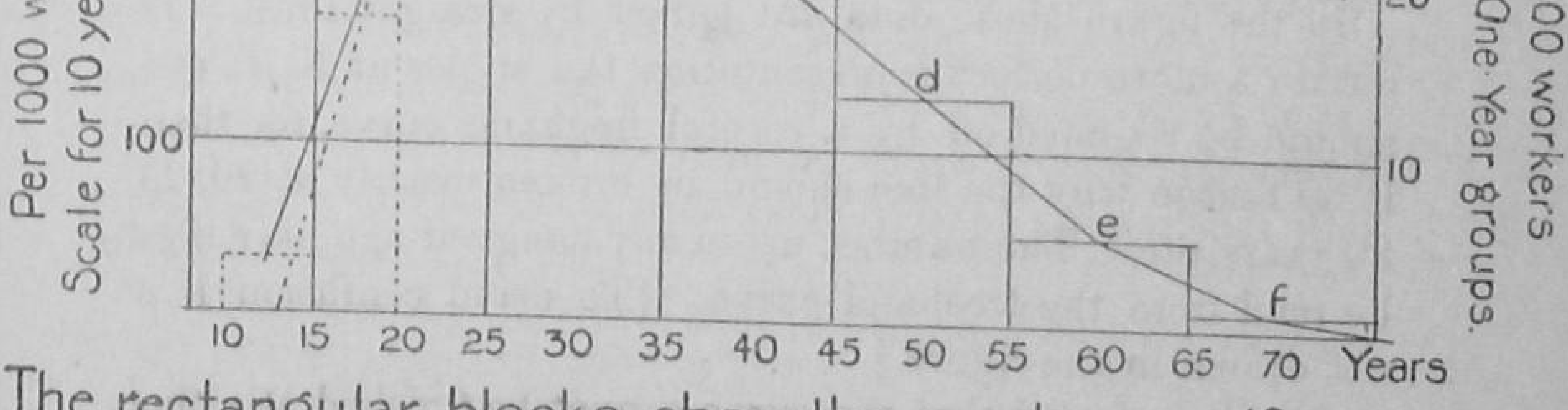
II

Ages of Coal-miners.

A Cumulative diagram showing the total number whose age is under each age marked on the horizontal scale:— e.g. 804 (per 1000) shown as PN, are under 45 years.



B Diagram showing distribution by Age.



The rectangular blocks show the numbers in 10 year groups.
The dotted rectangles show the five-year groups from 10 to 25 years.
The curve shows the number at each year.

Since there can be no sharp division of numbers as we pass from age to age, the apparent division being introduced by the accidental placing of the age limits, it is clear that the whole group should be represented by an unbroken line. The ordinary introduction of such a line as *abcdef* is intended for this purpose. A little reflection will, however, show that we should keep the area standing on any given base unchanged, and that this line cuts off a great part of the area on 25—35 years. To avoid this a freehand curve should be carefully drawn, so as to keep all the areas unchanged. It will at once be seen that in this case the information is not sufficient for an accurate drawing, and that there is something arbitrary in the figure. If it is found that the line is not definitely placed, the figure should be left in the original rectangles.

Finally, the extremities of the curve, below 14 and above 70 years, must be drawn to satisfy the conditions of the data (in this case there are no children under 12 or 13 years), the continuity of the drawing being preserved.

The vertical scale adds very little to the information, and might in this case be removed after the drawing is complete with little loss.

These difficulties are present to some extent in all group diagrams.

3. Diagram A represents the cumulative numbers in the last column of the table, p. 36. The dots (K, L, P, etc.) show the information exactly as it is given, and there is no element of approximation or arbitrariness.

In the figure these dots are joined by straight lines. To obtain a more perfect representation the angles at K, L, etc., should be rounded off by a careful freehand curve, for there is no reason why the line should be broken exactly at 20, 25, 30 years, etc. The number up to any assigned age may then be read from the freehand curve. [To avoid confusion it is not drawn in the figure.]

It will be found that the curve cannot be finished at either end without further information.

The quartiles and median (see p. 24) of the group may readily be found approximately from the drawing. The line MC is drawn horizontally through the middle point of the vertical scale to meet the freehand curve at C; CD is then drawn vertically to meet the horizontal scale at D. The reading at D (32 years) is the median.

The mode is the reading above which such a curve is steepest, but is not easily determined with accuracy by the eye, and needs mathematical analysis before an exact value can be obtained.

A diagram of this kind is more accurate and useful than such as B, and is more easily used for the comparison of two groups. It requires practice to grasp its meaning readily.

4. The final test of the goodness of a diagram is its legibility and clearness of meaning.* The diagram should carry on its face a sufficient definition of the facts represented. There should never be many lines in one diagram, unless they can be kept apart from each other. Lines should be distinguished by colours or clear hachetting, and, where suitable their meaning (*e.g.* "weight" and "value," p. 44) should be, written close to them. Cross references should be avoided. If there is much detail, either the data should be separated into two or more diagrams, or the numbers should be left in a table and not represented graphically. An overloaded diagram defeats the only purpose for which it is intended.

Any diagram can be drawn on the back of a postage stamp or enlarged to cover a wall. The page of a book is generally sufficient for all the detail that ought to be shown, and large sheets and folded pages are to be avoided. The *ratio* of the vertical to the horizontal scale must be chosen so as to bring out those fluctuations or movements which are the subject of study; then the absolute scale should be so chosen that the allotted space is occupied.

5. The following diagrams show the method of representing

* Diagram B above is, of course, only intended to show the method of working. The other diagrams printed in this book satisfy, it is hoped, the conditions here laid down.

	Average Annual Gazette Price of Wheat per Quarter.	Quinquennial Averages.		Differences.
1864	40·2s.	—	—	
1865	41·8s.	—	—	
1866	49·9s.	1864–1868	52·0s.	− 2·1s.
1867	64·4s.	1865–1869	53·6s.	+ 10·8s.
1868	63·7s.	1866–1870	54·6s.	+ 9·1s.
1869	48·2s.	1867–1871	56·0s.	− 7·8s.
1870	46·8s.	1868–1872	54·5s.	− 7·7s.
1871	56·7s.	1869–1873	53·5s.	+ 3·2s.
1872	57·0s.	1870–1874	55·0s.	+ 2·0s.
1873	58·7s.	1871–1875	54·7s.	+ 4·0s.
1874	55·7s.	1872–1876	52·6s.	+ 3·1s.
1875	45·2s.	1873–1877	52·5s.	− 7·3s.
1876	46·2s.	1874–1878	50·0s.	− 3·8s.
1877	56·7s.	1875–1879	47·7s.	+ 9·0s.
1878	46·4s.	1876–1880	47·5s.	1·1s.
1879	43·8s.	1877–1881	47·3s.	− 3·5s.
1880	44·3s.	1878–1882	45·0s.	− 0·7s.
1881	45·3s.	1879–1883	44·0s.	+ 1·3s.
1882	45·1s.	1880–1884	42·4s.	+ 2·7s.
1883	41·6s.	1881–1885	40·1s.	+ 1·5s.
1884	35·7s.	1882–1886	37·2s.	− 1·5s.
1885	32·8s.	1883–1887	34·7s.	− 1·9s.
1886	31·0s.	1884–1888	32·8s.	− 1·8s.
1887	32·5s.	1885–1889	31·6s.	+ 0·9s.
1888	31·8s.	1886–1890	31·4s.	+ 0·4s.
1889	29·7s.	1887–1891	32·6s.	− 2·9s.
1890	31·9s.	1888–1892	32·1s.	− 0·2s.
1891	37·0s.	1889–1893	31·0s.	+ 6·0s.
1892	30·2s.	1890–1894	29·6s.	+ 0·6s.
1893	26·3s.	1891–1895	27·9s.	− 1·6s.
1894	22·8s.	1892–1896	25·7s.	− 2·9s.
1895	23·1s.	1893–1897	25·7s.	− 2·6s.
1896	26·2s.	1894–1898	27·2s.	− 1·0s.
1897	30·2s.	1895–1899	27·8s.	+ 2·4s.
1898	34·0s.	1896–1900	28·6s.	+ 5·4s.
1899	25·7s.	1897–1901	28·7s.	− 3·0s.
1900	26·9s.	1898–1902	28·3s.	− 1·4s.
1901	26·7s.	1899–1903	26·8s.	− 0·1s.
1902	28·1s.	1900–1904	27·3s.	+ 0·8s.
1903	26·7s.	1901–1905	27·9s.	− 1·2s.
1904	28·3s.	1902–1906	28·2s.	+ 0·1s.
1905	29·7s.	1903–1907	28·7s.	+ 1·0s.
1906	28·2s.	—	—	
1907	30·6s.	—	—	

a *series*. In a series we have generally to study both the short-period fluctuations (regular or irregular) and the general movement or tendency, or "trend," as it may be called. In Diagram A (p. 42) the jagged line shows the data as given. It is at once clear that we have a succession of rapid fluctuations combined with a general movement mainly downwards. The problem is to disentangle the "trend" from the fluctuations. The table on p. 40 shows how it may be done.

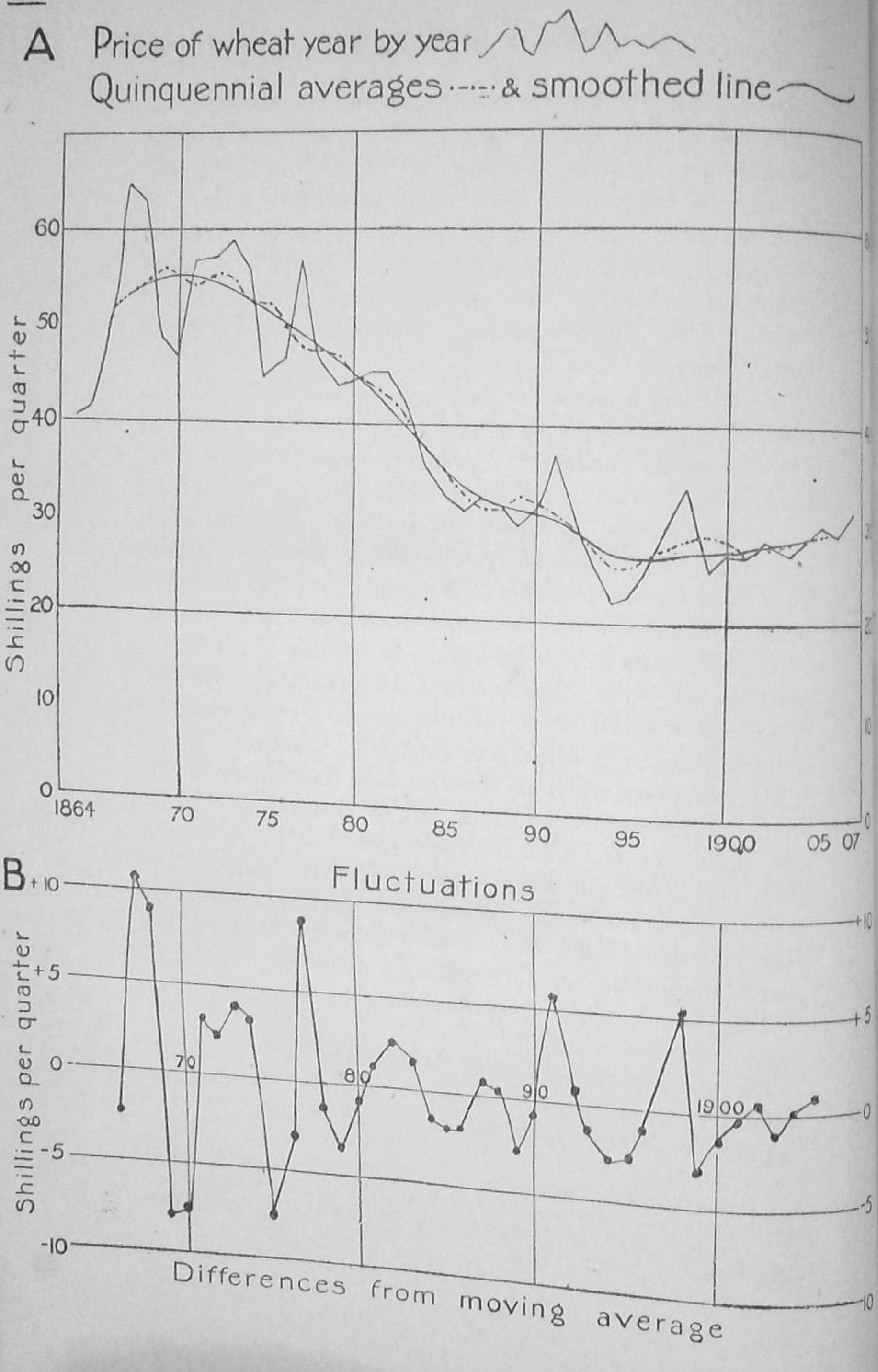
A period is selected, long enough to remove the fluctuations of separate years, short enough to allow a long series of averages to be obtained.* As in the second column of the table, averages are taken again and again, and the line of "moving average" is shown in the diagram. The angles and small fluctuations of this line should then be smoothed away, as they are accidental. This smoothed line shows the trend; in this case it is downward from about 1870 to about 1895, and nearly neutral with some inclination to rise in the most recent years. This line cannot begin at the beginning or end at the end, of the period covered by the data, for several years are necessary to establish "the trend."

It is now assumed that the smoothed line represents the course of the events, as determined by slow-acting, cumulative influences, and that the deviations from it are due to short-period (or, in some cases, accidental) causes. The deviations, or differences between the price of a particular year and the average price of the five years of which that year is the middle, are obtained in the table and represented in Diagram B; a new vertical scale is taken to throw the fluctuations into relief.

The smoothed line of Diagram A and the line of Diagram B show the "trend" and the "fluctuations"; but it is advisable to preserve also the jagged line of the first diagram.

* If the fluctuations occupy the same length of time (*e.g.* 10 years), again and again, this period (10 years) should be taken for the successive averages. It is not necessary to use the same period throughout the series.

III



A Price of wheat year by year
Quinquennial averages & smoothed line
Shillings per quarter
60
50
40
30
20
10
0
1864
70
75
80
85
90
95
1900
05 07
B
Fluctuations
+10
+5
0
-5
-10
Shillings per quarter
70
80
90
1900
Differences from moving average

There is something arbitrary in Diagram B, since the magnitudes of the differences, and sometimes even their sign, depend on the length of the period taken for averaging.

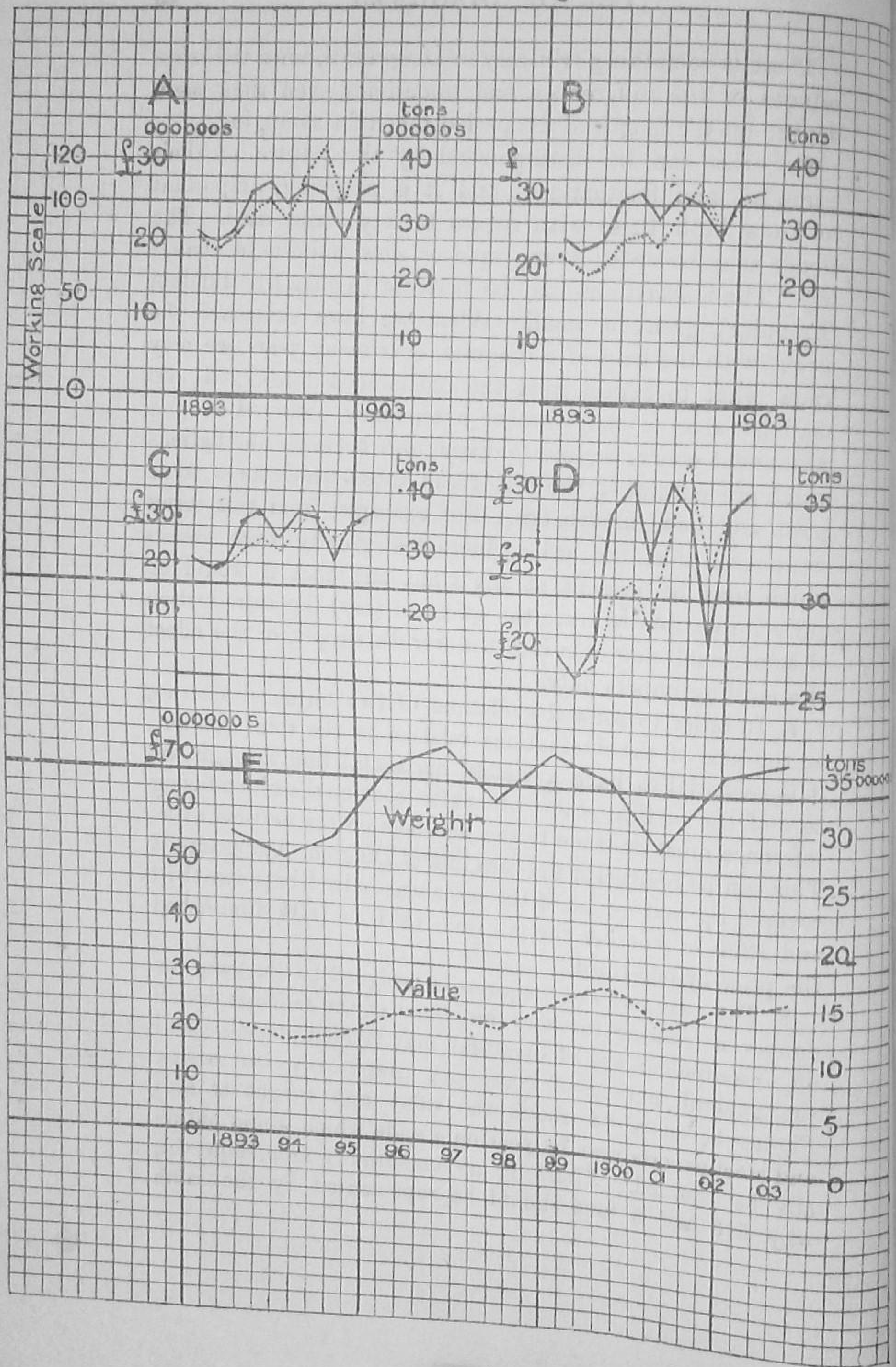
6. Series can be distinguished by the nature of their "trends" and fluctuations; and it is extremely important to know both these with regard to any series used. The trends may be up or down, rapid or slow, uniform or changing. The fluctuations may be periodic (regular) or random (irregular), great or small. When we have examined the series, with the help of a diagram, over many years, we may know what to expect from the phenomena considered; we shall be able to tell whether a tendency observed is of a permanent character, and to distinguish between fluctuations which are natural to the series and those which show some great and new disturbance. For example, from this series we notice that the price fluctuates greatly from causes which may be present at any time, and that it would be quite impossible to trace the effect of (say) the introduction of some small new area into the world's supply, or the effect of a shilling import duty.

7. In order to show the relations of two terms of a series, or the size of the fluctuations relative to the total amount, it is essential to have a visible horizontal line through the zero of the vertical scale; otherwise continual and confusing reference must be made to the numbers on the vertical scale. This can be realized if the zero line of Diagram A opposite is covered up, and we ask if the price was halved between 1870 and 1890, or whether the fall from 1891 to 1894 diminished the price by one-third.

8. The next series of diagrams is designed to illustrate the danger of ignoring the zero line, some of the fallacies which an unscrupulous use of diagrams may render plausible, and the general method of comparing graphically two series. Figure E, on p. 44, shows the value and weight of iron and steel exports year by year on a scale which would naturally be adopted. There is no essential reason, however, why £10 and 5 tons should be represented by the same

IV Exports of Iron & Steel Manufactures

Weight —— Value ....



vertical distance. In the three figures A, B, C, the weight is represented on a uniform scale, viz. half that of E; but in A the scale for value is so chosen that the lines begin together, and also (as it happens) the averages for the eleven years of value and of weight are represented at very nearly

EXPORTS OF IRON AND STEEL AND MANUFACTURES THEREOF, PRODUCE OF UNITED KINGDOM

			Relative Numbers.		Re'ative Numbers.			
			A		B		C	
Years.	Value £. 0000's	Weight. Tons. 000's.	Value	Weight.	Value	Weight.	Value.	Weight.
1893	20,26	2,738	81	81	72	81	49·4 + 0	81 + 0
1894	18,47	2,566	74	76	65	76	− 4·3	− 5
1895	19,43	2,738	78	81	69	81	− 1·8	+ 0
1896	23,46	3,423	94	102	83	102	+ 8·0	+ 21
1897	24,41	3,599	98	107	86	107	+ 10·4	+ 26
1898	22,39	3,160	90	94	79	94	+ 5·1	+ 13
1899	27,71	3,601	111	107	98	107	+ 18·4	+ 26
1900	31,62	3,447	126	103	111	103	+ 27·4	+ 22
1901	25,01	2,813	100	84	88	84	+ 11·6	+ 3
1902	28,88	3,474	116	104	102	104	+ 21·4	+ 23
1903	30,40	3,565	122	106	106	106	+ 25·0	+ 25
Average	£24,73	3,193	99	95				

the same height. In B the equation is made for the last year, 1903.* Both these are correct, but method B very frequently gives the better perspective for two series. In long series it is best not to equate individual years, but to equate the averages of the last few years given.

C and D are misleading; the lines for value and weight are accurate separately, but the zeros of the vertical scales are not in the same position. It is a simple arithmetical

* To obtain the working figures for A, take 81 (a number convenient for numerical work in this case) to represent the value in 1893, and obtain proportionate numbers for the other years with a slide rule or otherwise. Take the same number to represent the weight in 1893, and finish the column by proportion. In this case easy arithmetic is obtained by multiplying the value by 4 and the weight by 3 less about 1%. For B the same weight numbers are used, but the value in 1903 is equated to 106.

problem, of which part of the working is given in the table above,* to force the lines to begin and end together. D is merely C enlarged vertically.

A comparison of these five diagrams shows that almost any appearance may be given to fluctuations by a deliberate choice of scales, and suggests the need of care and intelligence in reading diagrams.

9. The following diagram shows one of the few methods of pictorial work that can be recommended. The proportion of the parts of a group to each other and the whole are shown by the sectors of a circle; since the areas of sectors are proportional to their angles at the centre and the arcs on which they stand, there is no possibility of confusing linear and areal proportions. For the comparison of two groups, two circles are constructed so that their areas are in the ratio of the numbers in the groups. It is at once clear by comparing the angles that the proportion (*e.g.*) of males between 14 and 15 years is smaller than that of

NUMBER AND AGES OF PERSONS OCCUPIED IN THE TEXTILE TRADES OF ENGLAND AND WALES (INCLUDING DEALERS), 1901

Ages.	Males.		Females.	
	Number.	Relative No.	Number.	Relative No.
10–14	24,700	18°	30,367	16°
14–15	18,332	13	31,402	17
15–20	81,200	59	188,125	102
20–45	267,168	196	359,976	196
45 and over	100,775	74	53,352	29
	492,175	360°	663,222	360°

$\pi r^2 = 4{\cdot}92175$; hence $r = 1{\cdot}252$ inches (1 sq. in. represents 100,000 persons),

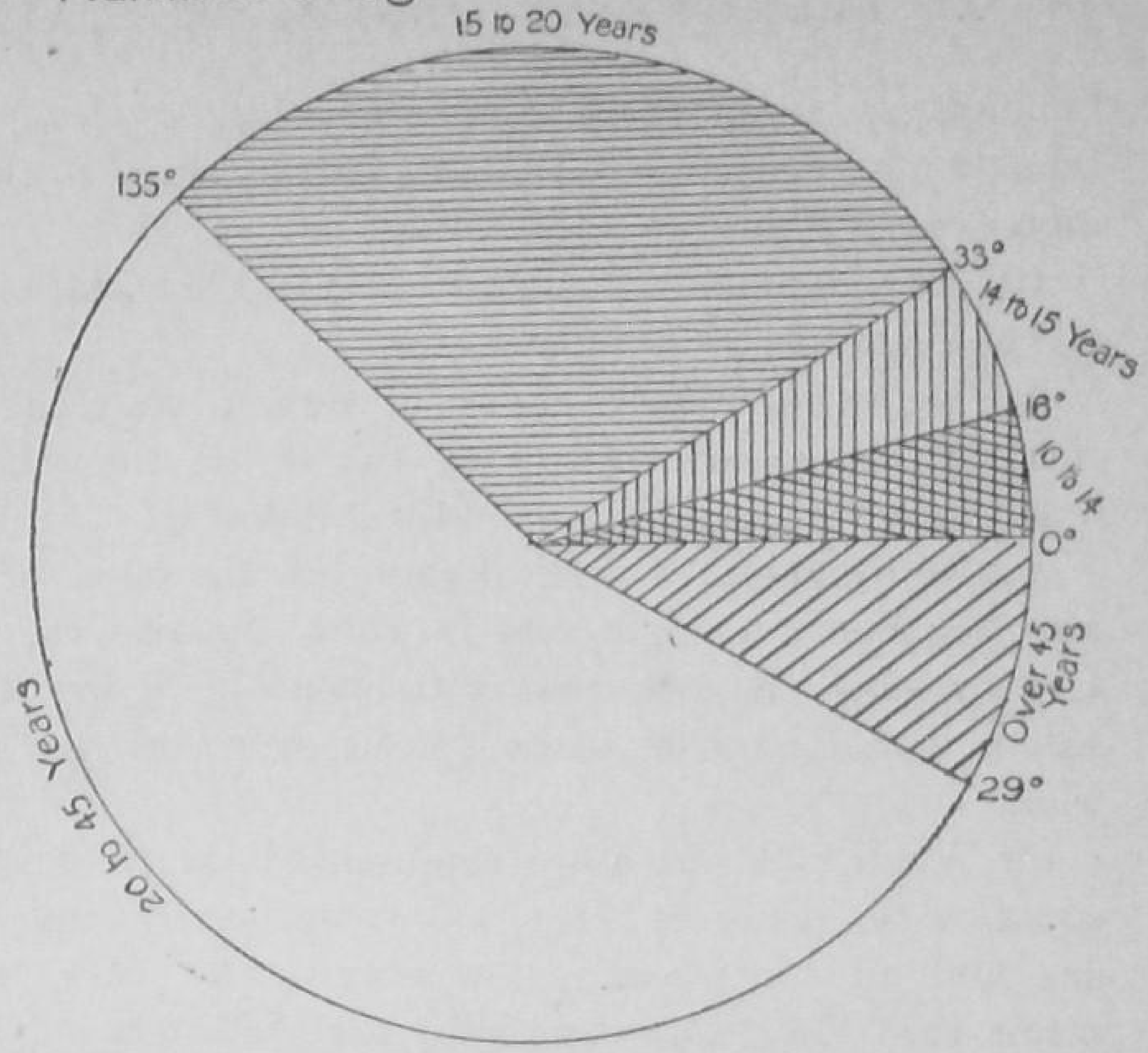
$\pi r^2 = 6{\cdot}63222$; hence $r = 1{\cdot}453$ inches (1 sq. in. represents 100,000 persons),

where r stands for the radius of the circle in each case.

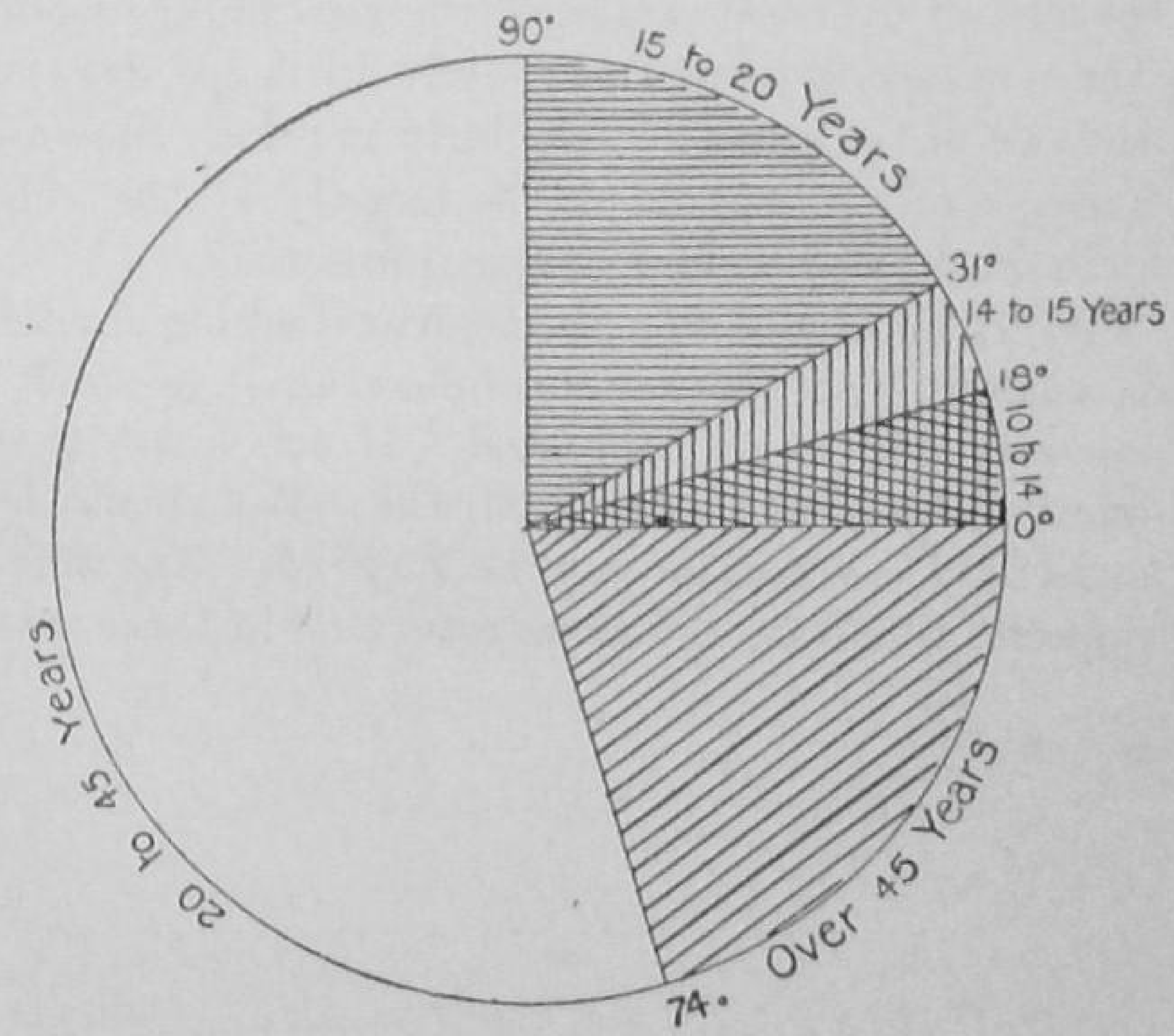
* The first and last figures for weight in A differ by 25. Equate the difference between the first and last figures for value (viz. 122 − 81 = 41) to 25, and reduce all the value numbers to the ratio 41 : 25; the first becomes 49·4, the last 74·4. Hence the numbers in the table.

V

Number & ages of Textile operatives



Females



Males

1 square inch represents 100,000 persons.

females; and observation of the areas suggests, (*e.g.*) that the number of women 20—45 years is about equal to that of all men over 20 years.

10. The commoner mistakes made in the construction and use of diagrams are as follow—

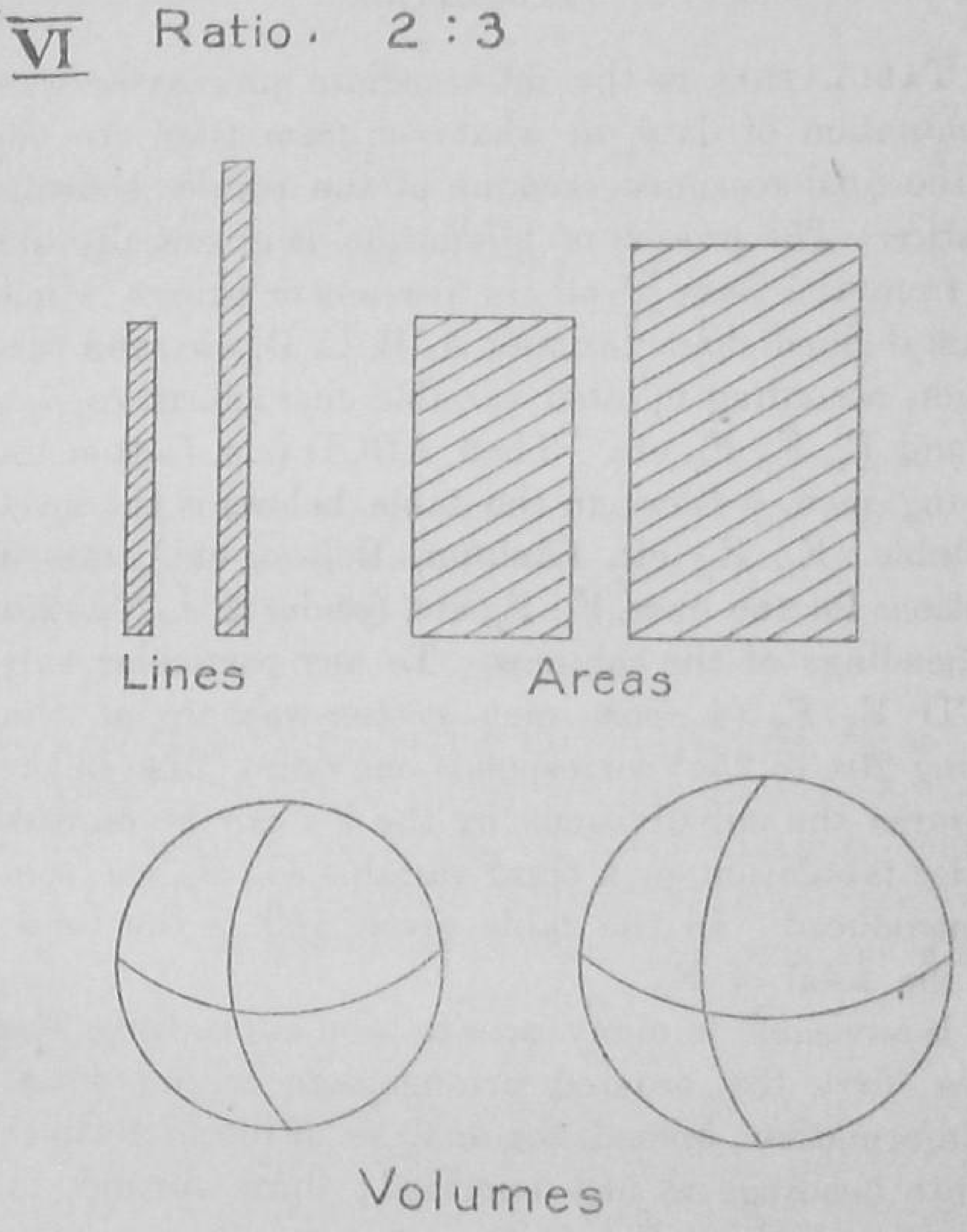
(*a*) By an injudicious choice of vertical scale the fluctuations are exaggerated (D, page 44), or, on the other hand, made inconspicuous (E, page 44, value line).

(*b*) An exaggerated vertical scale has the effect of making too conspicuous a single year in which the rise was greatest. It may easily happen that with monthly figures the high values would be seen to be spread over both the adjacent years.

(*c*) When two series are represented on one diagram the equation is made between an exceptionally high year in one and an exceptionally low year in the other, with the result that the relative growths are distorted.

(*d*) It is not always realized that in such diagrams as B (p. 42) and E (p. 44), the dot representing the number is to be placed over the *centre* of the horizontal distance showing the corresponding period; while in A (p. 37) the dot is at the *end* of the period. Similarly the dots showing a moving average (A, p. 42) should be exactly at the centres of the periods for which the average is taken.

(*e*) In pictorial diagrams (such as the "big and little loaf") it is seldom clear whether the linear, areal, or cubic dimensions are intended to be compared. If one quantity is $1\frac{1}{2}$ times another, for linear comparison the ratios should be 1·5 : 1, for areal 1·225 : 1, and for volume 1·145 : 1. The three diagrams opposite illustrate the same ratio 2 : 3 in three ways.


VI Ratio. 2 : 3
Lines
Areas
Volumes

CHAPTER VI

TABULATION

1. TABULATION is the intermediate process between the accumulation of data, in whatever form they are obtained, and the final reasoned account of the results shown by the statistics. The process of tabulation is essentially the selection from the data of all the persons or things, which have certain defined characteristics A, B, C, D, etc., and their subdivision according to other variable characteristics E_1, E_2, E_3, etc., and F_1, F_2, F_3, etc. Then ABCD (*e.g.* Cotton industry, weaving, men, 4 *looms*, in the table below) is the heading of the table; E_1, E_2, etc. (Ashton, Bolton, etc.), are the descriptions for the lines, F_1, F_2, etc. (under 20*s.*, 20*s.*–25*s.*, etc.), the headings of the columns. To any particular sub-group ABCD E_3 F_2 (4 *loom* men cotton-weavers at Stockport earning 20*s.* to 25*s.*) corresponds one entry (214) in the table. Of course the sub-divisions by the F's can be omitted for a simpler tabulation, or a third variable, G_1, G_2, can sometimes be introduced. In the table given 109 is the total of E_1, 799 the total of F_1.

It is advisable in many cases to tabulate in three successive stages: first, the ordered arrangement in full detail of all the information; second, the analysis of the first tables under definite headings as just described; third, abstract tables of the main results. The first set are merely for reference, if minute details may be wanted, or if further analysis may at some time be needed; the third set is a mere abbreviation of the second. In this chapter we deal with the second set.

2. The following table from the Reports on Earnings, etc., in the Textile Trades,* will serve to illustrate the discussion.

* Cd. 4545, p. 63.

Cotton Industry—Weaving

Number of Men Weavers (4 looms) working full time, whose Net Earnings in the last pay-week of September 1906 fell within the undermentioned limits.

Districts.	Under 20*s.*	20*s.* and under 25*s.*	25*s.* and under 30*s.*	30*s.* and under 35*s.*	35*s.* and above.	Total number.	Average earnings *s.* *d.*
Ashton-under-Lyne	9	67	29	4	—	*109*	23 10
Bolton	1	9	20	—	—	*30*	24 10
Stockport. . . .	10	214	75	—	—	*299*	23 3
Preston	37	406	361	23	6	*833*	24 10
Blackburn . . .	69	1,293	1,669	121	14	*3,166*	25 5
Accrington . . .	20	190	127	6	6	*349*	24 6
Burnley	185	1,448	1,942	402	86	*4,063*	25 11
Bacup	88	606	203	33	19	*949*	24 4
Rochdale	258	756	416	72	12	*1,514*	23 4
Other districts . .	122	535	147	6	1	*811*	22 7
TOTAL	*799*	*5,524*	*4,989*	*667*	*144*	*12,123*	24 11
Percentages . .	6·5	45·6	41·2	5·5	1·2	100	—

This is an example of double tabulation with cross totals. The problems isolated for study are the distribution of the number of weavers according to their earnings, and the variation of this distribution from district to district. It forms one of a series of tables in which the variation of wages according to occupation and district is examined.

3. Before making a table we must consider in detail exactly what information is wanted. The data generally consist of one or more items of information about each of many individual persons or things. In this case we know the industry, district, occupation, sex, age (whether adult or not), earnings, and length of time worked, for each person. We can group any three of these data in a double table. Here we take as the main heading the composite datum "industry, occupation, sex, age, and length of time (*i.e.* full time, $55\frac{1}{2}$ hours)," and tabulate according to the remaining two, viz. district and earnings. We might equally well tabulate

district and occupation, or occupation and earnings, or sex and earnings, etc. The result of the particular tabulation used is to show that earnings are nearly uniform district by district, and are concentrated in the two groups 20*s.* to 25*s.*, 25*s.* to 30*s.*

Where one of the quantities varies grade by grade (as wages, age, etc.) it is entered in the horizontal heading. The number of grades entered separately is limited by the nature of the material and by the consideration that the whole must be easily visible at once.

The order of the districts, or other terms, in the vertical list should be alphabetical if there is no natural order; but it frequently happens that there is a natural or geographical grouping which is of assistance in studying the relationships, or in making subordinate totals. Similar places or things should be next each other.*

The line of totals shows the distribution of wages in the occupation as a whole; the column of totals shows the distribution of the occupation among the districts.

Supplementary information can be added, if the table is not overcrowded. A percentage line, as the last, is often very useful. The column of average earnings makes the visualization of the figures easier.

In printing, great care is necessary to bring out the principal words in the heading by suitable type; and wherever there is a change in the significance of the numbers a change of type should be made. A typewriter is not capable of producing a good table.

4. The table should, if possible, show on its face its exact meaning. It is too commonly the case that a table can only be understood by a cumbrous system of notes or references or by searching through a great deal of preliminary matter. For this reason the heading is rightly long and carefully worded. If necessary the heading should be broken up into

* In the table just given the order of places is that used throughout the report, and is convenient for cross reference. It is partly geographical, partly according to the nature of the trade in the district.

a series of sentences, with great care as to space and typing. When the matter in hand is extremely complicated, it is better to use a brief heading, and to place a full description of the meaning of the table and definitions of the terms used in print on the page opposite the table.

It often happens that many of the entries require special explanation. These may be given by a series of notes legibly printed immediately under the table. References by *, †, ‡, §, ¶, etc., should be avoided if possible. Every one has suffered from the system of notes used in railway time-tables. In the case before us, the only further definitions wanted are those of the districts and of the distinction between men and boys. The former is given by reference to a page where the delimitation of districts is stated once for all the tables; for the latter one has to search through the introduction to the report to find that males over 20 years of age are counted as men.

It is generally the case that, however minute the tabulation, there is a residuum; here we have "other districts," and earnings "35*s.* and above." The residua should be made small compared with the total, and should be inserted to avoid confusion.

After a table is made it is often the case that it has to be re-cast to fit the printed page. Folded tables should be avoided; if the table is too big for a page, or for two pages facing each other, it should be split up in two or more. The eye cannot grasp more detail at once than will cover two pages.

5. A table should neither contain numbers consisting of many digits nor many blank spaces. The latter can be avoided by merging the unimportant lines in the residuum. The former will be avoided if careful attention is paid to the substance of Chapter II above. Numbers have very seldom more than a superficial accuracy beyond the third or fourth significant figures, and it is seldom that greater accuracy is required, unless for further numerical work. Large numbers in a table confuse the eye, destroy the legibility of the whole

and conceal the significance of the grouping; the wood is hidden by the trees. Either round numbers should be used, in such a way that the last digit printed is accurate, or the lines can be given as percentages or per thousands. It is to be remembered that full details are supposed to exist for reference in an earlier series of tables (not necessarily printed).

6. So far we have been considering the form and nature of tables intended to give public information and resulting from a collection of statistical data. Tabulation has further important uses. When an investigation as to any facts is made, it may happen that the groups, or classes, or series which result are predetermined in form, and that we have merely to fill in details in tables already prepared; but it frequently happens that we are in the position of an explorer, and do not know even what kind of things we may discover. In such cases the process of tabulation is the process of analysis. In the investigation as to wages in the cotton industry, for example, tables were made to determine how far the number of looms tended per person influenced wages, what was the relation between the earnings of spinners and of their piecers, whether wages were nearly at a uniform level from place to place, and many other such questions. For analysis of this kind the rule is simple; determine exactly what it is that is to be tested, devise the table that will answer the particular question and no other, fill in the details from the data, and perform the necessary arithmetic for any comparison wanted

7. Again, in considering the progress of an institution or a business, analysis is constantly wanted, and is carried out in tabular form. We deal with this subject in Chapter IX.

8. Diagrams, averages and tabulation can all be used for presenting the results of a statistical accumulation. Of these the tabulation is the essential. Diagrams only give the results of tabulations in a special form, suitable for showing the relations between the various numbers and for allowing a *coup d'œil* over the whole field, but they cannot replace the actual figures for purposes requiring minute accuracy or for

further numerical work; also, as stated above, they should only be used over a limited field, while the tabular form is universal. Averages are abbreviations, replacing the more complete table for purposes of comparison with other tables. The reduction of a column of figures to an average throws away a great part of the data. Much attention has been given in recent times to curing this defect of averages, but after all refinements have been made we cannot dispense with the details of the group averaged, and these are to be found in tables.

9. It seemed inexpedient to load this manual with many examples of tables; in Part II many small tables are given, but they should be regarded as the final kind of tabulation, *i.e.* "abstract tables of the main results." The reader can find innumerable examples in statistical publications, and should criticize them by asking the questions: "Are the headings intelligible? Are the terms used in the heading and the table sufficiently defined? Is important information omitted or unimportant included? Is the spacing and arrangement of type satisfactory? Is there any difficulty in picking out the essential information?"

CHAPTER VII

SAMPLING

1. It is not always necessary to obtain complete information as to all members of a group, in order to give an adequate account of it. Most practical judgments are formed by experience of a limited number of examples. Purchases are frequently made after examination of a sample. The satisfactoriness of a consignment of goods is tested by examining and testing a few bars, cases, packages, etc. The probable yield of a mine is estimated by assaying a small quantity of ore. The goodness of a water supply is ascertained by bacteriological examination of a microscopic quantity. Such methods are not only means of saving time and expense, but are absolutely necessary in some cases; for testing often destroys the commodity, as when a tin is opened or the breaking-strain of a steel bar is determined, and it is often impracticable to examine every part, *e. g.* in the case of a mine, whose contents is not completely known till it is exhausted.

2. The first essential of an examination by sample is that every member of the group considered should have nearly the same chance of being included in the sample. This may be secured either by mixture or by random selection.

Mixture.—Suppose it to be required to assay the quantity of gold in several barrels of the sweepings of the Mint, or the quantity of alcohol in many cases of wine, to take two eminently practical examples. In the first case, extract equal small quantities of dust from near the top, the middle and the bottom of each barrel. Mix each sample thoroughly, take an equal fraction of each and mix (say) four together; repeat this process of mixing and division till a quantity small enough to be assayed is obtained. In all such processes the methods of choice, mixing and division will be directed to neutralizing any physical irregularities of weight, shape, etc., which might

destroy the random nature of selection. To determine how nearly the result is correct, the process should be repeated (say) four times; the true result may be expected to be within the divergencies shown by the four measurements.

3. *Random selection.*—This is often sufficiently secured by the process of spreading out the consignment of goods, etc., and marking one taken here and another there, avoiding the first and the last and the most obvious, and testing the objects marked. Another method is to divide the objects into equal groups and take one at random from each group. The more scientific way is to secure absolutely equal chances by numbering the whole group consecutively, writing down the numbers on tickets and shuffling them, and finally drawing at random some of the tickets and examining the objects with the corresponding numbers. To avoid the writing and drawing, digits are sometimes selected at random from mathematical tables and used as if they were numbers drawn at random.

As before, the exactness of the result (if it is a case of measurement) should be tested by repeating the process, varying the selection each time.

4. In carrying out the above processes successfully in social or other investigation, less concrete than the examination of a consignment of goods, the first step is the careful and exact definition of the group to be tested. If, for example, we are examining the physical condition of school children, we should delimitate the area to be taken, enumerate all the schools in it, and find the number of children on the register of each; the group taken would then be co-extensive with the "registered school children." In making the measurements we should have to take children absent from school as well as present, if they happen to be chosen by the selective process used, as otherwise we should be taking the smaller group "children present at school"; this might give an imperfect result, as the absent children might contain a large proportion of the physically unfit. In any case, the group as described would not contain children removed from the district and specially treated in institutions.

The temptation is always to measure the obvious and

easily accessible; but if we do this our sample is of "the accessible," not of the whole group. Thus the budgets of working-class expenditure, which are often published, are not typical of the working class as a whole, but of that part of it which is intelligent enough to have some kind of record and is willing to communicate private details. In particular, the expenditure on drink is under-estimated.

5. *Determining the average.*—It is clear from common-sense principles that the larger the number included in the sample measurement, other things being equal, the more accurately the average will be determined; in Chapter IV it was stated that the precision increased as the square root of the number taken. This accuracy does not depend in any way on the *size of the group* from which the sample is selected; the average height of all the men in England can be determined with the same accuracy by the same number of measurements as the average in one town, if in each case every person has the same chance of inclusion. The following examples illustrate the increase of precision as more samples are included, and other points—

(*a*) Forty groups of ten entries each were taken at random from a list of the rate of interest paid by 3,878 Companies.

The average rates obtained for these forty groups were as follows—

AVERAGES OF 10 COMPANIES SELECTED AT RANDOM

Rate of Dividend.				Number of Occurrences.
	£	*s.*	*d.*	
Above	5	0	0	1
	4	18	6	3
	4	17	0	5
	4	15	6	7
	4	14	0	6
	4	13	0	8
	4	11	6	7
	4	10	0	3

The average of the 400 Companies, contained in the 40 groups, is £4 14*s.* 11*d.*

The original entries vary from 0 to £103 %. The averages of 10 are all between £4 10*s.* and £5 1*s.* It is then practically certain that the average of all is between these limits, and not far from the average of the 40 groups, viz.

£4 14*s.* 11*d.** Actually it is found to be £4 15*s.* 7*d.*, when all the Companies are included.

(*b*) A large number of packs of playing cards were mixed together, and 32 groups of 3 cards were drawn, and the number of pips on each were counted, Knave, Queen, King being taken as 0. The following was the result—

Total number of pips on 3 cards in order of drawing.	Total on 12 cards.	Average per card.	Total on 24 cards.	Average per card.	Total on 48 cards.	Average per card.	Total and average for 96 cards.
5	56	4·7	121	5·0	206	4·29	402 Average 4·19
16							
17							
18							
14	65	5·4					
19							
24							
8							
8	40	3·3	85	3·5			
5							
8							
19							
9	45	3·75					
10							
18							
8							
0	35	2·9	100	4·2	196	4·08	
11							
15							
9							
17	65	5·4					
18							
14							
16							
10	46	3·8	96	4·0			
7							
7							
22							
22	50	4·2					
17							
5							
6							

* These figures are given and more refined measurements are made in the *Statistical Journal*, 1906, pp. 550–3.

The original cards vary from 0 to 10; the averages of 3 from 0 to 8, of 12 from 2·9 to 5·4, of 24 from 3·5 to 5·0. It is then practically certain from the sample that the average of all is between (say) 3·5 and 5·0, and that 4·19 is a good approximation. Actually there are 55 pips to a suit of 13 cards (picture-cards counting blank), and the average is 4·23.

6. While the determination of the average is of great practical importance for purposes of valuing the group and other arithmetical work, it is often equally important to determine the proportion of various kinds in a group, as for example the number of families per 1,000 whose income is less than £1 per week, or the number of children per 1,000 suffering from remedial throat complaints. The following examples show a method that can be followed—

(*a*) The 400 Companies in the former example were divided into 4 groups of 100 each and tabulated according to the rate of dividend paid.

Rate of Dividend.	Number of Companies. 1st 100	2nd 100	3rd 100	4th 100	Together.	Per cent. Estimate.	Per cent. Actual.
Nil	6	5	8	9	28	7	6·0
£1 and under 3	3	0	3	0	6	1·5	1·5
£3 " 4	34	23	29	22	108	27	27·2
£4 " 5	25	30	28	34	117	29·25	31·1
£5 " 6	13	18	16	13	60	15	17·7
£6 " 8	9	16	9	14	48	12	10·8
£8 and above	10	8	7	8	33	7·25	5·7
	100	100	100	100	400	100	100

The last column but one shows the distribution as estimated from the sample of 400; the first 4 columns show how far the estimate can be trusted. Thus it is practically certain that rather more than half the Companies paid between £3 and £5, the numbers only varying in the 4 groups from 53 to 59. The number between £1 and £3 is doubtful. The last line, containing the exceptionally high dividends, is *a priori* uncertain; the accident of sampling may easily include too

many or too few rare cases. The method can only be trusted for the large, central divisions. The last column shows the actual distribution of the 3,878 Companies, from which the samples were taken.*

(*b*) In the draw of 91 cards (including all but the last five of the previous paragraph), the actual occurrence of the various numbers was—

Ace	8	
2	8	20
3	5	
4	7	
5	7	23
6	8	
7	8	
8	4	19
9	9	
10	6	
Knave	7	21
Queen	7	
King	7	

If the drawing had been continued, of course, we should have found less and less relative difference between the numbers. Here we have no actual test of the accuracy of the result.

[A mathematical way of dealing with such a question as "how many picture-cards are there in the given group?" is as follows: Let n be the number in the group, let m be drawn, and pm prove to be pictures. Then pn is the most probable number of pictures in the pack, but it is as likely as not to differ from this number by as much as $\frac{2n}{3} \cdot \sqrt{\frac{p(1-p)}{m}}$. It is very unlikely to differ by as much as six times the expression just written.

In the card experiment, if n were 1 820 (the number of cards actually in the group used), $m = 91$, $pm = 21$. The

* For the *a priori* test of accuracy, see again *Stat. Journal*, 1906, p. 553.

forecast from the sample as to the number of pictures among the 1,820 cards would be—

$$pn \pm \frac{2 \times 1820}{3} \sqrt{\frac{21 \times 70}{91 \times 91 \times 91}} = 420 \pm 54,$$

and we should feel sure that the number was between $420 \pm 6 \times 54 = 420 \pm 324$.

Per 13 cards we should have (by proportion) $3 \pm \cdot 4$,* and the maximum possible would be about 5.

Thus the experiment is not sufficient to determinate the proportion of picture-cards accurately. More cards would need to be drawn till the *m* in the above formula was sufficiently increased.]

7. No formal rules can replace judgment and experience in the selection and interpretation of samples. The simplest practical direction is to continue to increase the number of samples, till successive tests show sufficiently similar results. When dealing frequently with the same kind, of course, experience would soon show how many tests were sufficient.

8. Two other methods of sample measurement are sometimes used.

Suppose we wish to test the knowledge of a large class of students (say 100). We might by some very simple examination, or by consulting the teacher, place them roughly in order of intelligence, and then examine in detail, say Nos. 1, 10, 25, 50, 75, 90, 100 (the maximum and minimum, median, quartiles and two deciles).† Thus a good estimate could quickly be obtained, and the relative ability of two similar classes quickly judged.

In the same way we could describe any group that can be placed in order, by the detailed examination of a few *selected by rule*. This method differs essentially from the method of random selection already explained.

* Observe that this result is independent of *n*.

† The deciles are the values which divide a group into ten equal parts, in the same way as the quartiles divide it into 4. If these seven positions are determined quantitatively for any group, a diagram of the form A, p. 37, can be drawn with considerable accuracy.

9. Rather than trust to the arbitrary action of chance, some investigators prefer to choose what they believe to be typical groups, and examine them in detail. Thus, investigations as to the wages, etc., of agricultural labourers have been conducted by selecting some forty districts throughout the country, so as to include types of all kinds of agriculture, and of all economic situations. This method results in an accurate and intelligible picture, but there is no easy means of calculating any average, or of knowing the distribution by number of persons earning various rates of wages. For filling in details where the general results are known the method is to be recommended.

CHAPTER VIII

RULES FOR USING PUBLISHED STATISTICS

1. It is never safe to take published statistics at their face value, without knowing their meaning and limitations, and it is always necessary to criticize arguments that are based on them, unless one is able to trust implicitly the knowledge and good faith of the persons bringing them forward. It is extremely easy to falsify the lessons which numerical statements should teach. The actual use and appreciation of statistics is ultimately a matter of intelligence, special knowledge and common-sense; but the following nine rules suggest the lines of study and criticism.

First.—Find the exact definition of the units which go to make the total. What is a soldier? What one pound's worth of exports? What a registered birth? What a member of the population, a case of fever, a bushel of wheat? One of the standard questions in agricultural statistics is "What is a cow?" In every case the definition depends on the regulations and method of collection. Thus we need to know at what stage a recruit is entered as "on the strength of the regiment"; what goods are counted as exports, and how they are valued; whether all births are registered, and whether still-births are included; how travellers, absentees and the homeless are counted; what are the rules for diagnosis of fever; whether wheat is weighed or measured; when a heifer grows into a cow, and much more detail of this sort. Generally expert knowledge is needed; sometimes the report on the statistics contains sufficient explanation and definition; sometimes the whole can be worked out from a study of the blank forms of

inquiry (with instructions) on which the original data are obtained.

The apparent meaning of a total is seldom its real meaning, but generally results from an artificial definition, necessitated by the process of collection.

As examples may be suggested the discovery of what is meant (i) by a room, (ii) by a farmer, in the census reports.

2. *Second.*—Consider how far the persons or things grouped together in a total or sub-total are similar; in other words, how far the group is homogeneous. Thus, persons whose occupations are grouped under the main heading "Textile Fabrics" differ with respect to (1) sex, (2) age, (3) nature of the material worked (cotton, wool, etc.), (4) position in the industry as merchant, dealer, manufacturer, or employé, (5) specific occupation, (6) locality. If we are merely told that 1,155,397 persons were included under the main heading in England and Wales in 1901, the information is so wide as to be nearly useless. An example of the most minutely defined group given in the census reports is:—County Borough of Oldham: number of males between the ages of 25 and 45 engaged in spinning process in cotton was 2,711. To know the meaning of this we should have to go carefully through a spinning-mill.

Whether the group or sub-group is sufficiently homogeneous depends entirely on the purpose for which the figures are used. If we compare the total numbers in the cotton industry in 1891 and 1901 we should be misled, because the numbers of children, men and women are in quite different proportions at the two dates; but a useful comparison might be made between the numbers of men.

The possibility of change in the relations shown when the groups are analyzed into parts of greater homogeneity must always be borne in mind. Innumerable examples might be given; an important one arises from death-rates. The rate is calculated by dividing the number of deaths in a district in a year by the number of persons living in the district

midway through the year, and multiplying by 1,000. Analysis at once shows that the various age-groups of the population are subject to quite different risks of death, and that the risks differ also according to sex; further, deaths from accident, from infectious diseases and from other causes should be in different categories.

If the internal constitutions of two groups are the same, *e.g.* if the distribution by age and sex are the same, then averages based on them may be properly compared. But we must never assume either homogeneity or similarity of division without knowledge.

3. *Third.*—Having defined and analyzed the totals, the next question is what is the relation of the quantity they measure to the quantity as to which we want knowledge. We wish to know the stress of unemployment, we learn the number of trade-unionists out of work; or of poverty, and we are told the number in receipt of public relief; or we are examining the improvement in health of the population, and we find the amount of disease and the number of deaths; for education we can tell the number of students, or of student-hours, or of examination successes. These statistical totals and averages are at best indices ,not actual measurements, of the more subtle and often incommensurable quantity or quality, which is essentially the object of the investigation. In order that indices may be useful they must at least move up and down with the quantity they represent, as the thermometer moves with heat and the barometer with pressure, and they should further make great or small oscillations with great or small movements; but many of them have less relation to the complete phenomena than the thermometer has to sensation of heat (which depends also on moisture and physiological conditions), and may be as remotely connected as the fall of the index of a barometer with the fall of rain.

If experience shows that the indices are sensitive and trustworthy they may be used to bridge over the gap between one more complete measurement and the next.

4. *Fourth.*—Before trusting or even reading a statistical

account, it is well to sit down and think quietly what statistics ought to have been collected, if possible, for the purpose in hand, and what sources of information exist, or should exist. Thus, if wages were to be measured, we should decide that the weekly rate, the annual earnings allowing for unemployment, supplementary earnings and the earnings of other members of a man's family should be known, and that allowance should be made for any necessary expenses; further, the money value should be interpreted in purchasing power, and the standard of life attained should be clearly shown. Of these things some it is not possible to measure; we cannot measure the actual satisfaction obtained from the expenditure of money, nor the value of unpaid personal work. Others, as the annual receipts and complete expenditure, could only be measured if the persons concerned kept accurate accounts.

Having got so far, we may take up the statistical report and consider how far the problem has been understood, whether all the practicable measurements have been made, and whether the result gives a true index in the sense of the last paragraph. We can thus decide as to whether the information is sufficient for solving any assigned problem; in only too many cases we find that it is not.

Further, if there is any suspicion of bias, of the intention to support any preconceived view, the criticism of method must be particularly rigid, and the maximum possible effect of the unconsidered factors must be allowed for.

5. *Fifth.*—When we have to deal with averages, rates, and percentages, we must carry our second rule of criticism further. Not only must we consider whether the numerators and denominators are homogeneous in themselves, but whether the terms of the denominator have a reasonable relation to those of the numerator. Should, for example, the number of deaths from small-pox be counted in relation to the whole population, to the vaccinated population, or to the number who contract the disease? Should the birth-rate be reckoned per 1,000 of the population or per 1,000 of married couples? Should the production of coal per head be reckoned

with respect to the population of a nation, or to those engaged in the coal trade, or only to the coal-hewers? The general answer is that the denominator should be limited to those who have a direct relation to the numerator; the legitimate birth-rate, *e.g.* should be in relation to married couples with some restriction of age. It may happen that this restricted denominator has a constant relation to a larger population, and in that case the latter may be used for simplicity of working (sometimes for lack of the detailed information), and for comparison with similar averages. Thus the number of births (929,807) in 1901 in England and Wales may be stated as 160 per 1,000 married women or as 28·3 per 1,000 persons; this last results from the combination of the two rates, 160 births per 1,000 married women and 177 married women per 1,000 persons. If the 177 remained unchanged, the two rates 160 and 28·5 would of course have a constant ratio to each other.

6. *Sixth.*—When two quantities are compared we must consider whether they are strictly comparable, and for this purpose most of the foregoing rules are necessary. Comparisons are made between two similar measurements at different dates (*e. g.* population, death-rate, average wage, production of wheat, etc.), or between two similar measurements relating to different places (*e. g.* trade, consumption of meat or wheat per head, amount of taxation per head, total or average income in two countries, etc.). We must test whether the two measurements are made on the same basis, so as to be indices of the same kind of phenomena considered, so as to cover the same ground and suffer from similar "error of bias" (see pp. 29, 32). Having ascertained this and so used rules 1 and 3, we then apply rules 2, 4, and 5 if necessary.

By such means we shall readily realize the difficulty of minute comparisons over long periods, during which relations have continually changed, and the extreme roughness of comparisons between such measurements as the indices of prosperity of two nations. Accurate comparisons can only be made between closely similar things or over quite short periods.

7. *Seventh.*—Closely connected with the last is the measurement of accuracy. In Chapters II and IV the approximate nature of statistical measurement was discussed, and some methods were given of testing the accuracy of results. In all statistics we must decide whether the data and methods will yield results accurate enough for the arguments based on them. It would be absurd to speak of an increase in average wages from 20*s.* 3*d.* to 20*s.* 6*d.* in twelve years, for the average could not be determined to 1*d.* in either case, and the group considered would have changed its character in the period; but we could speak reasonably of an increase of "about 50 %" if the averages were 20*s.* and 30*s.* The less the groups satisfy the stringent conditions of the first six rules laid down, the greater must be the margin allowed for error. Where possible, the greatest possible errors arising from imperfection of data or processes should be worked out.

8. *Eighth.*—We must not depend on figures relating to single days, months, or years, or on comparisons relating to short isolated periods. In Chapter V the fact that every measurable recurring phenomenon yields a series of definite characteristics was illustrated. These characteristics, the natures of the fluctuations and of the trend, must be known In the case of the population of a large country, where there is little emigration or immigration, it is not difficult to fill in estimates for intermediate years; in the case of the total value of exported goods it is impossible. Every measurement must be viewed in the light given by a series of similar measurements stretching back over a long period; otherwise temporary fluctuations will be taken for permanent changes, as if a cold summer were regarded as proving a change in climate; or a rise will be reported, when the whole trend is downwards, as if we should compare the bank holiday traffic of a decadent tramway one year with the lowest day's record of this preceding.

Where a sufficient record cannot be obtained, judgment must be suspended.

9. *Ninth.*—Having determined as far as possible the exact

purport and limitations of the statistics, consider (without reference to the printed report) to what conclusions they lead, or whether they are so imperfect that no conclusions can be reached without further investigation. There is often a great gap between the statistical table and the non-statistical conclusions that are fathered on to it, especially if the statistics were obtained in order to support a preconceived theory. Statistical work properly ends with such a dull, colourless, matter-of-fact report as is customary in the publications of the British Government. As a separate process such results are to be taken in conjunction with non-statistical knowledge. Inferences are suggested and tested by the reported facts, and a severely critical and logical analysis is necessary before the whole investigation leads on to some reasoned action.

CHAPTER IX

METHODS OF STATISTICAL ANALYSIS

1. In the previous chapter the way to criticize statistical reports was outlined; in this chapter we consider briefly the methods of collecting statistics at first hand, (i) for the purpose of testing the progress of a commercial undertaking, (ii) for testing the success of an institution, (iii) for collecting data for the solution of a social problem.

2. Details vary so greatly for different kinds of business, that it is only possible to lay down some general principles with illustrations. The processes of book-keeping and accountancy are, in their more refined forms, examples of statistical investigation, and, so far as £ *s.* *d.* is concerned, provide the data, even if they do not give the result, of such analysis. When accountancy is applied to commodities as well as to money, we arrive at statistics. Take the case of wool-spinning. The data that should be tabulated are—the weight and cost of raw wool used in a given time, in the aggregate, in each room, and by each mule; the weight of yarn produced, in similar detail, and the weight of waste material recovered; the price realized for the products (or, if the yarn is used in the same factory, the estimated value); and the cost in wages and in oil and sundries. Over a longer period an estimate should be made for the interest on the capital value and the depreciation of the machinery used, together with a proportional allowance for the general expenses of the factory, such as salaries, rent and rates, and advertising. The cost of the engine should be placed under the special expenses, if possible; if not, this cost must be divided between the various rooms with what accuracy is practicable. With such data it is possible to tell what

machines, rooms, or departments are running at a loss, or just paying their special expenses, or contributing adequately to the general expenses, or making a profit.

In this case, also, it is easy to state the number of lbs. of wool spun and the length of the yarn produced, and the actual work done by each group of operatives (the spinner and piecers at each pair of mules), which is, in fact, measured for the basis of piece-wages. It can be at once determined whether the machine (the spinning-mule) is being used efficiently.

Similarly in weaving, data are easily available for the product per loom, per operative, and per £ of wages paid, and the totals can be made for each weaving-shed and for the factory as a whole.

3. A more complicated problem is presented in railway working, and an example of the method of compiling statistics in use on many important railways in America, India and elsewhere is very instructive. The data are twofold, based respectively on the details of the train service, and on the quantity of goods conveyed; the first are connected with expense, the second with remunerative work done.

For each journey of each train the guard sends in a report as to the time at which the engine arrived, the times (actual and due) at which the train arrived at and left each stopping point, and as to the number of wagons (empty or loaded) hauled each section of the journey, with other details. For each journey of each engine the driver reports the time he was working with the engine, and how it was allotted to standing, shunting, or running, the amount of coal taken on, and the number of the wagons hauled in each section of the route.

On the other side, returns are made of all consignments of goods, showing the stations at which they were received and where they were delivered, and the money received for their transit.

From these data the following tables among others are compiled:—

STATISTICS OF OPERATION. GOODS AND MINERALS TOGETHER.
Months of June 1904 and 1905.

	No. of working-days.	Tons carried.		Ton-miles.	Average train-load.	Train-miles.	Average distance hauled.		
		Total.	Per working-day.				Goods.	Minerals.	Together.
		000's	000's	00000's	Tons	000's	Miles	Miles	Miles
1904	26	4468	172	1026	100·3	1023	35·4	18·5	23·0
1905	26	4325	166	980	104·5	938	34·3	18·6	22·7

	Engine-hours.			Ton-miles per engine-hour.	Train-miles per train engine-hour.	Train-miles per single track-mile per working-day.	Ton-miles per route-mile per working-day.
	Train.	Shunting.	Total.				
	00's	00's	000's				
1904	1386	1922	331	310	7·38	13·2	2363
1905	1263	1821	308	318	7·43	11·8	2223

	Wagon-miles.				Average train-load of wagons.			Wagon-miles per engine-hour.			Average wagon-loads.
	Loaded.	Empty.	Total.	% Loaded of Total.	Loaded.	Empty.	Total.	Train per hour.	Shunting per hour.	Together per hour.	
	0000's	0000's	0000's								Tons.
1904	1939	1174	3113	62·3	18·9	11·5	30·4	224	162	94·1	5·29
1905	1821	1123	2944	61·9	19·4	12·0	31·4	233	162	95·4	5·38

"Ton-miles" form the principal measurement of the revenue-yielding work done by a railway so far as freight is concerned, and are obtained by multiplying the number of tons in each consignment by the number of miles it is carried. "Train-miles" signifies the aggregate of the miles run by trains; "engine-hours" the aggregate of the hours in which an engine was working with a train, distinguishing running from shunting. The total of wagon-miles is computed by multiplying the number of wagons moved by the number of miles run separately for every section at the beginning of which the composition of a train was altered,

and adding these products. These results, together with the total of the tons moved and the fixed information as to the track, are sufficient for the tables.

Let T be number of tons moved, T_m number of ton-miles, T_{rm} number of train-miles, E_t and E_s numbers of train and shunting engine-hours, W_l and W_e the number of loaded and empty wagon-miles. Then the average train-load is $\frac{T_m}{T_{rm}}$ tons; the average distance hauled is $\frac{T_m}{T}$; the average of ton-miles per engine-hour is $\frac{T_m}{E_t + E_s}$; of train-miles per train engine-hour is $\frac{T_{rm}}{E_t}$; the average train-load is $\frac{W_l + W}{T_{rm}}$ wagons; the average wagon-load is $\frac{T_m}{W_l}$, and the average number of wagon-miles per engine-hour is $\frac{W_l + W_e}{E_t + E_s}$.

Such figures could be worked out for any division of the railway that is required. By comparing the averages obtained for different months or different divisions, we can observe the work done by engines in hauling goods or wagons (ton-miles or wagon-miles per engine-hour), the use made of the track (or railway as ordinarily understood) and of double lines (train-miles per track-mile and ton-miles per route-mile), what proportion of haulage is effectively spent in hauling full wagons, and how heavily the wagons are loaded. Where any one of these averages increases, there is presumptive evidence of growing efficiency in working; where a difference or decrease is shown, there is a case for inquiry as to the cause; it may prove to be due either to the nature of the work, or to incompetency in handling it, or to a reorganization which produces a compensatory improvement elsewhere.

From similar tables the receipts per ton, per ton-mile and per train-mile are worked out for different classes of traffic.

4. In the case of railways and other large undertakings the problem is to discover exactly what measurement is most sensitive to efficiency of work, and to devise the necessary machinery for obtaining the statistics of precisely that measurement. In the running of goods trains the principal expense that can be reduced is the time during which the wages of the three men (driver, fireman and guard) concerned are paid; "wagon-miles per engine-hour" and "ton-miles per engine-hour" are found to provide precisely the tests wanted. In other cases it might prove to be the production per spindle per week, or the output of coal per hewer. When such tests are devised and kept systematically, an instant indication is given of any improvement or slackening in the work, and the reasons of the change can then be investigated.

5. It is clear that such a broad average as "wagon-loads" obtained by dividing 103 million ton-miles by 20 million loaded-wagon-miles does not satisfy the test of homogeneity suggested above (p. 65); a railway may be engaged in hauling coal by the train-load and also in handling small parcels for quick delivery; for the former heavy wagon-loads are easily obtained, with the latter the rapidity (and the custom) may be lost if goods are not forwarded till a wagon-load is ready. In other industries high average production may depend on inferiority of goods. Where the relative proportions of the different classes of work done vary very little, this consideration will not vitiate the comparison of averages; but where the proportions are not steady, further analysis and subdivision must be made, so far as practicable, till statistics are obtained for nearly homogeneous work; the first step made in this direction in railway statistics is in separating minerals from other goods. In the same way the analysis should extend, both for quantity and cost, to the smallest subdivision of the work that can be separated.

The labour and expense of collecting statistics in this way is much diminished if, when the actual averages or quantities which form the most delicate tests of efficiency have been decided on, no statistics are accumulated, which are not

directly needed for these averages, etc., and simple printed forms are used, which can be easily filled in an ordinary routine; these forms should be regularly delivered to a statistical clerk, who should systematically tabulate them on a uniform scheme.

6. There are two considerations which affect the use and formation of such statistics; first, the value of money is subject to continuous changes; secondly, it is not easy to find a common measure of the work done.

For the change in the value of money the reader is referred to Part II, Chapter IV, below, with the suggestion that special index-numbers should be formed to suit particular circumstances.

The addition and comparison of unlike quantities can often be made by the device of "weighted totals." This can be illustrated by the general statistics of the worsted trade.

EXPORT OF WORSTED TISSUES

	1894. Yards.	1907. Yards.	Mean price, 1894–1907. d. per yd.	"Weights."	Numbers adjusted to common measure. 1894.	1907.
	0000's	0000's			00000's	00000's
Broad coatings, all-wool .	1117	1379	46·2	20	2234	2758
" " mixed .	385	720	28·0	12	462	864
Narrow coatings, all-wool .	217	41	31·7	14	304	57
" " mixed .	272	159	20·3	9	245	143
Stuffs, all-wool . . .	1320	1043	11·9	5	660	521
" mixed . . .	7756	6559	9·4	4	3102	2624
Total of Worsted Tissues	11067	9901			7007	6967
Ratio . . .	100 : 89·4					
Total value .	£6,666,000	£7,394,000		.	100 : 99·4	
Ratio . . .	100 : 110·9					

During this period (1894–1907) the price of wool and of woven tissues fluctuated considerably, 1907 being a year of high prices. The aggregate value is therefore not a fair measure for comparison. The value in four of the six categories into which the exports are divided fell, and rose in the other two. The aggregate yardage fell. Now, a yard of

"Broad pure wool coatings" cannot properly be added to a yard of "mixed stuff"; the first is much heavier, broader and more expensive than the latter. The average prices of these six classes are shown in the table; the first is worth five times as much as the last per yard. Assume that these prices are proportional to the intrinsic values of (or to the work done in producing) the cloth, and for simplicity of computation take integers nearly in the proportion shown. These are called "weights" in the sense of Chapter III above, and it is known (pp. 18, 32) that they need not be taken with great accuracy for purposes of comparison. Multiply the quantities by the "weights," and so obtain the last two columns; here in effect the unit is "one quarter of a yard of mixed stuff" equal to $\frac{1}{20}$th of a yard of pure broad coatings, etc.

The comparison of the weighted totals shows that the total production was practically the same in 1894 and 1907 on this basis, though the value rose 11 % and the aggregate yardage fell 11 %.

The actual average weights of wool per yard in the various classes might be used for the statistical weights if they could be estimated. This method is used in the railway statistics of live stock, when one horse is counted as equivalent to so many sheep or to so many fowls, for purposes of transit cost, and it is capable of wide and varied application.

7. It is often useful to make and keep up to date charts of prices, cost, output, wages, etc., in considerable detail. In particular, if a trade is seasonal, it is well to have a graphic record of the seasonal fluctuations, with a view to forecasting the immediate future, and to providing an adequate supply for the probable demand.

It is generally interesting and sometimes of importance to preserve a record of the rates of wages paid to various classes of operatives, and also the average for the whole. It has frequently proved to be the case that the average has risen faster than the rates, owing to the different growths of various grades of labour and to readjustments of work. Such changes are often unobserved, but are frequently the main

factors in the growth (or, less frequently, the diminution) of earnings.

8. The principles of measuring the progress and efficiency of an institution are similar to those just outlined, but the statistical aspect is less important; for, while a commercial company is in business for the dollars and the test of success is pecuniary, an institution exists for carrying out some defined aim, for which there is in general no numerical measurement. Nevertheless it is more necessary to test the statistics offered by the management of an institution, especially when it is appealing for help, than those collected by a commercial body for itself; for it is in the interest of the latter to know the facts exactly, while the former needs to show a good case, and there is nothing so easy as to show a biassed result without actually falsifying the facts. In its own interest, for success in working, an institution should record its facts on a commercial basis, and in candour should present these records to the section of the public concerned. Hospitals, asylums, schools, colleges, and propagandist, religious, philanthropic and social societies are among the institutions to which these remarks apply.

As regards £ *s. d.*, accounts should be kept in great detail and carefully allotted to services and departments. In particular the expenses of advertising, of collecting money, of printing, of postage, and of administration, should be shown clearly, and separated from the expenditure directly on the objects for which the institution exists; the former correspond to the general expenses of manufacture. Further, when building, new or old, is involved, the exact state of the building account should be shown, and the amount spent on rent, interest, rates and taxes. When these things cannot be found clearly in a balance-sheet, suspicion always may arise that there is something to conceal. The proportions of the foregoing expenditures to total expenditure afford tests of the efficiency of administration that, when applied with knowledge of what has been done in similar cases, are very useful.

The costs of carrying out the objects of the institution should then be allotted, so far as they can be properly credited, to a department or group of departments. Averages should then be worked out—for a hospital, the cost of food and other household expenditure per head per week; for a school, the cost of teaching per child per term; and so on. At this point the question of homogeneity must be considered. The averages just mentioned would be useful in an asylum or workhouse or general hospital usually nearly full, and in a large primary school, but not in an institution where there were many grades of expense, or a college where there was specialized teaching for small classes; nor would one judge a missionary society by its expenditure per convert. The less an institution belongs to a regular type, and the less uniform the persons it deals with, the less, also, can general averages be usefully applied; but where it is possible to compare like with like, then the causes of differences in such averages should be sought out.

9. As regards statistics of results, of success in carrying out the declared aim, it is well to apply Rule 4 of the previous chapter; think out what is the exact measurement that is wanted. In a hospital the number of patients dealt with, together with the average length of stay,* and details of the number cured or relieved, should be known; the number of operations is often stated, but it may include the extraction of a tooth as equal to tracheotomy, and is not of much use. In an asylum the number of persons should be given classified by sex, age and length of sojourn. In a teaching institution the difficulties are greater. No sensible person regards examination tests as adequate. The number of registered students is misleading, as the amount of time nominally given and the regularity of attendance vary greatly; the information should rather be given in details showing (for example) the numbers of students, subdivided by age and standard of instruction, the number of classes

* A railway statistician would probably ask for "patient-days per bed."

per week attended, and a measure of the regularity of attendance; also the size of the classes should be stated. In some cases total teaching-hours, total student-hours, and student-hours per teacher may be stated with advantage; but these are likely to be misleading and suggest resemblance between railways and the business of teaching, which would only be found in a very wooden educational scheme.

A more useful way of studying such statistics is to compare them in detail year by year, and to try to account for the differences shown, remembering that the smaller the numbers dealt with the more apparent will be the variation from causes that are fortuitous and independent of the management of the institution.

In the end, statistics of this kind can only help to form judgments, which should be based mainly on non-statistical observation.

10. Our final subject in this chapter is the collection of data in connection with some social inquiry—for example, the amount of unemployment, the physique of children, the condition of a district as to overcrowding, or the more elaborate investigations that have been made as to general social conditions in London, York, Dundee, West Ham and Birmingham. The first thing to do is to think out in a quiet hour exactly what we desire to know, and, next, what part of this knowledge can rest on a statistical basis. For unemployment we might decide that the essential thing to discover was the number of hours' work obtained in the previous month, for overcrowding the number of cubic feet in a tenement per occupant, and so on, but we should at once find that additional measurements were necessary—*e. g.* in the last case the ages and sex of the occupants and the condition of ventilation. At this stage it is best to work out blank tabulations, where each column, row and total would give definite information on the subject of inquiry; then work out forms of questions, the answers to which would lead to the tabulation desired. Next consider what persons possess the information required.

The construction of the blank form of inquiry on which the answers are to be entered depends on the education and position of the people who are to fill it in. In general, it is useless to issue blank circulars unless the filling them in is compulsory. If the information already exists in written form, *e.g.* the record of wages paid at a factory, it can frequently be obtained by a personal visit at which the object of the inquiry is briefly explained, and interest aroused or at any rate consent obtained; and then a blank schedule carrying a clear explanation of what is wanted on its face, can be left. The questions must be such as can be answered by "yes" or "no" or in numbers; adjectives such as "fair," "occasional," etc., are nearly useless for tabulation, their significance varies from person to person. If, on the other hand, the data must be collected first hand, a house-to-house visit may be necessary. The labour may be abbreviated if the method of samples (Chapter VII above) can be strictly applied. Of course, tact and experience is necessary for this work. A separate blank form, again containing perfectly definite questions, should be used for each case; but except where measurements are necessary, the answers should be obtained in conversation and entered immediately afterwards; for a visitor taking notes is likely to be an object of suspicion.

11. The data having been collected, their working-up can be done in the light of the previous chapters. The special difficulty in this kind of investigation is the essential indefiniteness of the quantities (poverty, physique, etc.) to be measured. It is well not to draw a single definite line, and say above this line is health, below it weakness, or above this mark competence, below poverty, but to remember that health, poverty, unemployment, overcrowding, etc., are relative. The final statistical table should be a graduation—so many tenements where there was more than 500 cubic feet per person,* so many at 400 to 500 cubic feet, and so on. Then

* In this case a person should mean an adult, and children should be counted as fractions according to their age.

the effect of drawing the line at various grades can be observed.

All statistics which cannot bear full criticism should be put aside, even if the inquiry has to be given up; imperfect statistics on such questions are often only productive of harm. In publication, the whole method of inquiry should be clearly and frankly shown, the tabulations should be perfectly clear, and the statistics of the inquiry be definitely separated from other parts, which deal (for example) with supposed causes and suggested remedies. Space should not be wasted in printing elaborate tables of data, but enough detail must be shown to allow a critic to form an accurate judgment as to the adequacy of the inquiry.

PART II

CHAPTER I

THE POPULATION CENSUS

1. THE population of the United Kingdom has been counted once in ten years; the first census was in 1801, the most recent is that of 1911; midnight before the first Monday in April has been the date taken in recent censuses. Blank forms are left with every householder, whose duty it is to enter certain particulars as to every person dwelling in the house alive at midnight. Precautions are taken to avoid omissions and duplications, and persons not in houses are counted as far as possible. A supplementary test of population is afforded by the enumeration of the number of inhabited houses.

The population enumerated for a district is thus the number who happened to be there at a particular moment, which differs from the number who live there habitually and differs greatly in many important cases from the number who work there. The accidental element arising from absence on journeys or presence on visits is not important in most cases; but it is evident that the population of holiday resorts fluctuates greatly through the year, and that the selection of April is arbitrary.

The principal questions put to every person are as to age, sex, condition as to marriage (known as "civil condition"), number of children, occupation, and birth-place. The number of rooms occupied by the family group is stated. The Royal Statistical Society is continually pressing for a more frequent census and for improvements in, and additions to, the questions asked.* Readers should compare the Census schedule

* See *Statistical Journal*, 1908, pp. 496–8, and 1909, pp. 574–593.

of 1911 with that of 1901, and follow the movement in favour of a permanent Census Office and more adequate information.

The organization of the census and the working out and publication of the results are entrusted to the three Registrar-Generals of England and Wales, Scotland and Ireland. The forms of questions and the methods of publication differ in the three countries. The principal general results for the United Kingdom are brought together in the General Report on the Census for England and Wales.*

Preliminary reports stating the population and number of houses, in all the subdivisions dealt with by the census larger than the parishes, are published within a few weeks of the date of taking the census. This was followed for England and Wales during 1901–3 by a series of volumes dealing with one county at a time †; next came summary tables,‡ bringing the figures for the various counties and districts together, dated April 1903, and finally, and not till July 1904, the General Report comparing the results and methods with those of the previous censuses, and containing important tables relating to special questions.

The Scotch Census for 1901 was published in three volumes; the first, dated July 1902, dealt with Houses and Population; the second (February 1903), with Ages, Birth-places, Civil Condition and Education; and the third (August 1903), was devoted to Occupations.§

For Ireland, separate parts are issued for each county, separate volumes for each of the four provinces, and a general report for the country; for the 1901 census these were all published before the end of 1902. The general report shows the size of agricultural holdings in considerable detail.

We were without any general occupational statistics relating to the United Kingdom, more recent than those of April

* Cd. 2174. Price 2*s.* 8*d.* Pp. 302–315.

† There is also a volume relating to islands in the British seas.

‡ Cd. 1523. Price 2*s.* 6*d.* This is the volume most useful for general reference.

§ For changes in the Census of 1911 see Appendix III, p. 213.

1901, till the year 1914, and great practical reforms, such as those connected with Labour Exchanges and Insurance, had to be initiated without adequate statistical basis.

2. There is very great confusion and complication between the various subdivisions * of the United Kingdom, and the boundaries are very frequently re-adjusted. The main tabulation of the census relates to "registration counties," of which there are forty-two (including London) in England; Wales is divided into three: Monmouthshire, South Wales and North Wales. Each registration county is divided into districts (Lancashire, for example, contains 27), and these into sub-districts.

For administrative purposes England and Wales is otherwise divided into County Boroughs and Administrative Counties, and the latter into Municipal Boroughs, other Urban Districts and Rural Districts, and these into civil parishes. It is likewise divided into Parliamentary Counties and Boroughs, which form or are subdivided into constituencies. A further general division is into Ecclesiastical Provinces, Dioceses and parishes, and another into County Court circuits and districts. Each of these four methods of division is subject to continual alteration in detail, as the shifting of population requires.

To make confusion worse confounded there is also the division into ancient counties, as given in geography books, where London is cut up into parts of Middlesex, Surrey and Kent.

The census publications deal with all these divisions, record is preserved of all changes, and the general rule is adopted that for comparative purposes the population on the area as defined in the most recent census is stated alongside with the population on the same area (however it was formerly divided) in previous times. With each county volume is issued a map, showing as plainly as possible the separate divisions.

3. To make any detailed comparison for a district it is

* See note at end of chapter.

necessary to work carefully at the county volume. For example, consider the question, "In what way did the district commonly known as Hull, with its suburbs, change its character between 1891 and 1901?" The population of the present area of the County Borough of Kingston-upon-Hull is given for both years, but the boundary was slightly changed in 1897, and in any case is not coincident with the area affected by the neighbourhood of the great town. "The Registration District of Hull" (No. 521, in the county of York, East Riding), "is co-extensive with the civil parish of Holy Trinity and St. Mary." This district is divided into Humber and Myton, the boundaries of each of which were changed between 1891 and 1901. Next to Hull is the Registration District of Sculcoates, divided into eight sub-districts, seven of which contain parts of the County Borough of Kingston-upon-Hull, which latter contains the whole of the Registration District of Hull. The sub-districts are further divided into parishes, and there are many illuminating footnotes, such as "A considerable proportion of the population of Willerby Civil Parish was enumerated in Hull City Lunatic Asylum." On looking at the map we see that Sculcoates (No. 520), contains the purely rural division of Ferriby (population 1,845), and the Municipal Borough of Hedon (population 1,010).

For persons who do not know the district, the only method is to lump the two registration districts of Hull and Sculcoates together, subtract all the small districts named where the population is less than (say) two to the acre, and regard the residue as the population of what is generally known as Hull. This will give an adequate idea of the growth between 1891 and 1901, and for a single year can be made correct to within 5 per cent. Such estimates should, of course, be given in round numbers. Persons in the district, on the other hand, will find that the information published is intelligible and sufficient.

4. The preceding paragraph shows that the problem of tracing the growth of urban population is not to be undertaken by the beginner in statistics. As a matter of fact it

is almost impossible to define the difference between urban and rural population, except by the technical and almost accidental division into "urban districts" and "rural districts," according to their administrative powers; but many so-called urban districts are more rural in character than some so-called rural districts, which have not yet adopted urban systems of drainage, etc. The towns are constantly growing past old boundaries, and neighbouring villages gradually assume urban characteristics, while still in the midst of agricultural country.

5. The following table shows the growth of the population of England and Wales, Scotland and Ireland, separately and together. As an example of further analysis the populations of London and of the principal manufacturing counties are also shown. The county of London contains about 120 square miles, the great part, but by no means all, of which is thickly populated; adjacent to it are considerable districts which for many purposes should be included in London, and the population of London and its dependent environment may be put at rather more than seven millions. In the table the northern counties include the whole of Cheshire, Lancashire, Yorkshire (West Riding), Durham and Northumberland; the midland counties include the counties of Derby, Leicester, Nottingham, Northampton, Stafford, Warwick, Worcester, Monmouth and Glamorgan. It will be noticed that the population of each of the three selected regions doubled in the fifty years 1851 to 1901, while that of the rest of England and Wales increased only 50 per cent.

GROWTH OF POPULATION

	United Kingdom. 0000's	England and Wales. 0000's	Scotland. 000's	Ireland. 000's	Present County of London.	Mining and Manufacturing Counties. Northern.	Midland.	Rest of England and Wales.
					0000's			
1851 .	2737	1793	2889	6552	236	451	276	829
1861 .	2893	2007	3062	5799	281	529	321	876
1871 .	3148	2271	3360	5412	326	628	364	953
1881 .	3488	2597	3736	5175	383	757	427	1030
1891 .	3773	2900	4026	4705	422	862	489	1126
1901 .	4146	3253	4472	4458	454	976	568	1255
1911 .	4522	3607	4761	4390	452	1084	655	1417

6. Closely connected with the distribution by locality is the distribution by occupation. This classification is extremely difficult, and it is prudent to take only those comparisons which are given in the census volumes, and to regard even them with suspicion, unless one has time to go into the question in minute detail, reading the text of the General Report for each census, and studying the changes in classification.

In using the following table it must be realized that the figures are per 1,000 of the selected part of the population, not absolute numbers, and that one such division can only grow at the expense of another. For example, the actual number of females working in connection with Textile Fabrics was greater in 1901 than in 1881. Further, it must be remembered that the groups are not homogeneous (see p. 65 above) either in age or in occupation. The number of occupied children tends to diminish as educational requirements are enforced; this accounts, for example, for part of the diminution under the heading "agriculture." The table is greatly contracted, and only suggests broad outlines for investigation.

Under the heading "Professional," etc., are included those engaged in government, central or local, and their subordinates, the army and navy on land or in port, and members of the professions and their assistants. "Domestic" excludes gardeners and coachmen, but includes a growing number of laundry-workers, lift-attendants, etc. "Commercial" includes merchants, dealers, "travellers," and clerks. "Transport" includes railways (but not railway construction), roads, rivers, docks and the telegraph and telephone services. [By the grotesqueness of the census tabulation the Post Office comes under heading I, 1. "National Government."] "Metals" includes all work in metals, except mining, and the manufacture of tools, machinery and engines, ships and carriages. "Building" includes navvies and road labourers. Sailors and soldiers are only included in the census enumeration when on land or in port at or within a few days of the

date of the census. "Undefined" includes a diminishing number of agricultural and builders' labourers.

The residual heading in the census, "Without specified occupations or unoccupied," has no relation whatever to "unemployed"; it includes among the males in England and Wales 352,000 persons retired from business,* 84,500 pensioners, including old age pensioners, 52,000 "living on their own means," and 1,700,000 others, "including students." The number of women "without specified occupations" is of course very much greater.

GROUPS OF OCCUPATIONS IN THE UNITED KINGDOM

	Per 1,000 Males over 10 years.			Per 1,000 Females over 10 years.		
	1881.	1901.	1911.	1881.	1901.	1911.
Professional, etc.	46	52	59	17	23	26
Domestic	8	9	11	142	122	110
Commercial	30	41	45	1	5	9
Transport	75	95	97	1	1	2
Agriculture	188	136	125	16	9	6
Mining	49	60	70	—	—	—
Metals	75	91	97	3	4	5
Building	74	86	70	—	—	—
Textiles	48	38	39	61	52	51
Dress	35	32	30	59	54	48
Food and Lodging	54	60	62	15	22	29
Other Manufactures, etc.	61	74	77	11	16	18
Undefined	85	61	50	8	8	10
Total occupied	827	834	832	335	316	315
Retired or unoccupied	173	166	168	665	684	685
	1,000	1,000	1,000	1,000	1,000	1,000
Actual total number of persons over 10 years (0000's omitted)	1255	1554	1719	1350	1680	1856

7. Apart from the census we have other information as to numbers of persons actually employed in factories and workshops, in or about coal mines, and in some other occupations, from returns made by employers to inspectors or to the Home Office. The difference between the two sets of statistics affords a good example of the necessity of examining

* Other than the Army, Navy, Church or Medicine; these are tabulated under their professions.

definitions and methods of collection, and also of the necessary approximate character of all general estimates.

Number of persons occupied in	Census 1901, United Kingdom.	Home Office Statistics.
Cotton manufactures	545,959	522,623
Woollen and worsted manufactures .	239,843	259,909
Silk manufactures	37,459	31,555
Hosiery manufactures . . .	60,950	38,549
Lace manufactures	41,453	17,902
Other textile manufactures . . .	73,398	158,815
Coal mines	752,626	792,648

In the case of cotton and wool, practically all persons occupied work in mills or factories, and the difference between the two columns is to be partly accounted for by the number of factories in which both cotton and wool are used, and the inclusion or exclusion of supplementary processes (*e.g.* dyeing). There are many persons working in hosiery and lace outside factories; hence the difference shown in the numbers.

8. The areas of all the districts and sub-districts are stated in the county reports, and the density of the population (the numbers of persons per acre or per square mile) can be worked out in minute detail. It is important for this purpose to take sufficiently small areas, for the least consideration shows that for nearly all practical purposes the variation over the area of a square mile is more important than that from county to county.

The table on page 93 shows an analysis for the neighbourhood of Middlesbrough, as compared with England as a whole and in contrast to Shoreditch in particular.

In this case we have the crowded town of Middlesbrough, occupying 4 square miles, separated on one side from the municipal Borough of Thornaby, less crowded and occupying 3 square miles, by 17 square miles of rather empty agricultural country, while on the other side of it lies an area of 10½ square miles of urban district containing on the average about one

house to an acre. This great diversity would be entirely concealed if we had not analysed the registration district into its parts. Maps, which are shaded to show the density of population, are apt to be extremely misleading because the changes of colour can hardly be made over sufficiently small areas, and if they were, the great detail would be confusing.

	Area (including inland water). Acres.	1901.		
		Population.	Number of persons per square mile.	Number of acres per person.
England and Wales . .	37,327,479	32,527,843	558	1·15
Yorkshire (North Riding) (Registration County) .	1,277,104	375,918	188	3·40
Middlesbrough—				
Whole Registration District	22,254	139,773	4,022	·16
Contained Registration Sub-districts—				
1. Middlesbrough . .	7,480	92,668	7,929	·08
2. Ormesby . . .	6,786	30,326	2,860	·22
3. Thornaby . . .	7,988	16,779	1,344	·48
The same in detail—				
1. County Borough of Middlesbrough . .	2,685	91,302	21,760	·03
1. Linthorpe † . . .	515	417	516	1·24
1. Marton † . . .	3,121	819	168	3·81
1. West Acklam † . .	1,159	130	72	8·92
2. Eston Urban Districts .	2,453	11,199	2,922	·22
2. Normanby ,,	1,500	9,645	4,115	·16
2. Ormesby ,,	2,833	9,482	2,149	·30
3. Hemlington † . .	1,119	115	66	9·73
3. Ingleby Barwick † .	1,519	124	52	12·25
3. Maltby † . . .	1,117	139	80	8·00
3. Stainton † . . .	2,306	347	96	6·65
3. Thornaby-on-Tees (Municipal Borough)	1,927	16,054	5,332	·12
Rural District of Middlesbrough, containing parishes marked †	10,856	2,091	123	5·19
London—				
Shoreditch . . .	1,131	206,180	116,530	·005

9. The census affords a test of the amount of overcrowding, detail being given for every town, urban district and rural district. The test taken is the number of persons relatively to the number of rooms occupied, and tenements in which there are more than two persons to a room are considered to be overcrowded. Of course, this test is very rough, for overcrowding depends also on the size of the rooms and the age and sex of the persons, to name only two considerations, but it affords some means of measuring progress and of noticing where serious overcrowding is prevalent.

The following table shows part of the information for London, and for the aggregates of County Boroughs, of other urban districts, and of rural districts in England and Wales.

OVERCROWDING IN ENGLAND AND WALES, 1911

Number of tenements (00's throughout)

Tenements of	Occupied by	County of London.	County Boroughs.	Other Urban Districts.	Rural Districts.
1 room.	1 or 2 persons . .	1059	483	335	92
	More than 2 persons .	324	165	74	15
2 rooms.	4 or less persons . .	1494	1483	1449	828
	More than 4 persons .	470	393	333	154
3 rooms.	6 or less persons . .	1908	2980	2805	2082
	More than 6 persons .	283	434	362	224
4 rooms.	8 or less persons . .	1536	6012	6697	4750
	More than 8 persons .	111	252	282	173
5 rooms.	10 or less persons . .	919	4990	6436	3820
	More than 10 persons .	23	74	87	42
Total number of tenements* . .		10240	23675	27820	17697
Number of overcrowded tenements*		1216	1329	1150	614
Per cent. of tenements overcrowded*		11·9	5·6	4·1	3·5
Per cent. of population in overcrowded*		17·8	9·4	7·4	6·5

10. The census affords no complete means of distinguishing industrial rank; the director, manager, foreman, artizan and labourer are not distinguished as such, for the main lines of demarcation are between industry and industry, and the

* Including tenements of more than 5 rooms.

subordinate divisions are according to the general processes (the spinning- as distinguished from the carding-room), and not according to occupation (the spinner as distinguished from the piecer). Nor does it show the social grading. Mr. Booth (*London*, Final Volume, pp. 1–9) has made a very ingenious social classification of the families of London, according to the number of persons per room or rooms per person for the families who do not employ servants, and according to the number of persons per servant for those who do. The information was specially extracted from the census forms, and has not been obtained for the country as a whole.

11. The population in 1901 is of course equal to that of 1891, together with the number of births and immigrants between the census dates, less the number of deaths and emigrants. But the emigration statistics, even for the United Kingdom as a whole, were not till 1908 adequate for such estimates. We have rather to work backwards to find the net result of migration and travelling.

England and Wales.—

	000's
Population, 1891	29,003
Births, 1891–01	9,156
	38,159
Deaths, 1891–01	5,563
Population in 1901 if no migration . . .	32,596
Enumerated population	32,528
Deduced excess of emigrants over immigrants .	68

The excess of the number of births over deaths is called the "natural increase of population."

It is necessary for many purposes to estimate the population at intermediate dates. The most accurate method would probably be to make the best estimate possible from the migration statistics, whose effect can be checked every ten years, and combine these with the recorded numbers of births and deaths.

Another way is to assume that the population increases continually in geometric progression; this rate was equal to ·95 % per annum for the United Kingdom between 1891 and 1901, and to ·79 % between 1881 and 1891; it is clear that some process of "smoothing" is necessary to pass from one rate to the other in 1891.

The following table shows various methods of estimating the numbers for the United Kingdom at the middle of each year, 1891 to 1901.

		000's		
Population in 1891	. .	37,802	Logarithms . .	4·57751
1901	. .	41,551		4·61859
Excess	. .	3,749		10)·04108
				·004108

	Arithmetic progression. 000's	Geometric progression		Computed from Births and Deaths less 59,000 net emigrants annually. 000's	Official estimate. 000's
		Logarithms.	Numbers. 000's		
1892	38,177	4·58162	38,162	38,140	38,134
1893	38,552	4·58573	38,524	38,494	38,490
1894	38,927	4·58983	38,889	38,881	38,858
1895	39,302	4·59394	39,260	39,265	39,220
1896	39,676	4·59805	39,633	39,655	39,598
1897	40,051	4·60216	40,009	40,063	39,986
1898	40,426	4·60627	40,390	40,453	40,379
1899	40,801	4·61037	40,773	40,827	40,772
1900	41,176	4·61448	41,161	41,181	41,152

The first column assumes equal annual increments of 3,749,000 persons; the second method assumes an annual rate ·95 % (log 1·0095 = ·00411); for the third method the births and deaths in the United Kingdom are taken from the Statistical Abstract,* and it is calculated that net emigration must account for a loss of 590,000 in the decade, including about 520,000 from Scotland and Ireland. It is not stated how the official estimate is obtained.

The first method is the most rapid, and agrees with the others within 2 per 1,000. The more involved method, com-

* Actually to obtain the number of births between mid-1891 and mid-1892, half the total numbers for 1891 and 1892 together is taken. Similarly for deaths.

bining numbers of births and deaths and emigrants, is likely to be the most correct, if the migration figures are studied more minutely. In the decade considered difficulties arise from the absence of troops in South Africa.

Any of the above methods can, and one or other must, be used for estimating the inhabitants of a county district or town; the "natural" increase is known from registration, but here is grave risk of error due to migration. The difficulties are accentuated when we estimate the population in (say) 1909, before we have the census of 1911. We may, however, take the "natural" increase, and compare it with the increase that the previous intercensal rate of growth shows; we can base another estimate on the number of school children; and in some cases check the result from the number of houses rated, but this is difficult. If these four methods agree, our estimate is good; their disagreement is a measure of the inaccuracy of the result. Local knowledge will sometimes allow the better of the four estimates to be chosen.*

12. The previous paragraphs deal with only a few of the very large number of problems and results of interest that arise from the census volumes. In conclusion, we will deal very briefly with the statistics of age. Age is stated inaccurately for the very young (through misreading of the instructions), for the very old (through ignorance or through the desire to magnify old age), and by women who are unwilling to confess even under the cover of secrecy to advancing age, and generally there is a tendency to return the age at the nearest round number, instead of at the last birthday; this would be corrected if the *date* of birth were stated. There will in future no doubt be a tendency to overstate age, with the idea that an old age pension may depend on it.

To overcome the concentration at round numbers, ages are tabulated as between 25–35, 35–45, etc. The other mistakes

* Students who wish to study the methods in use should consult papers by Mr. Waters, p. 293, and by Mr. Hayward, p. 434 of the *Statistical Journal*, 1901, and follow up the references there given.

cannot be completely rectified, but they can be checked by two different methods. First, there is the record of persons at the various ages at all the previous censuses, and the registers of deaths according to age and of births; from these the number surviving can be estimated, and the differences found must be attributed to migration * or mis-statement of age; the tables (18 and 19 of General Report of Census) show the existence of some of the mis-statements already named, but in general confirm the accuracy of the answers. Secondly, it is certain that in a large population the numbers at successive ages must result in a continuous and regular group; there cannot be a great number at 30, and relatively few at 29 and 31. The application of this principle is the basis of the life table, the survival table, the tabulated death-rates according to ages, and the other tables which supply actuaries with material for their calculations. The method of smoothing in the diagram, p. 37, depends on the same idea. It is beyond our scope to discuss here the mathematical methods which are employed. The Census General Report gives the result of the "graduation" for the whole population year by year from 0 to 105 years (Tables 20 and 21). The following contracted table is important as showing the relative number of young and old, and of the two sexes

Number per 10,000 Persons Enumerated in England and Wales in 1901

	Males.	Females.
Under 5 years	570	572
5 and under 15	1,048	1,051
15 " 25	947	1,011
25 " 35	764	852
35 " 45	594	635
45 " 55	429	463
55 " 65	279	318
65 " 75	147	184
75 " 85	51	70
85 and over	6	9
	4,835	5,165

* Or temporary absence in the case of soldiers.

NOTE ON CERTAIN DIVISIONS according to which the population is tabulated in the Census of England and Wales.

Ancient Counties are the old counties, 40 in England, 12 in Wales, which have been only slightly changed in historical times by the merging of their detached parts in the counties by which these are surrounded; in the case of Worcestershire considerable parts are still detached.

London was constituted as a separate administrative county, carved out of Middlesex, Kent and Surrey, in 1888.

Registration Counties are groups of registration districts, covering to a great extent the same areas as the Ancient Counties by whose names they are called. The *registration districts* are simply the Poor Law Parishes and Unions utilized for registration purposes, births, marriages and deaths, as well as census enumeration and tabulation. In connection with the Poor Law Reforms of 1834 parishes were grouped into Unions for Poor Law purposes round convenient centres, and county boundaries were generally ignored. Consequently the groups of registration districts which form a registration county overlap the ancient county boundaries seriously; for example, the populations of the Ancient and the Registration County of Derbyshire were 620,000 and 490,000 respectively in 1901,* but in most cases the differences are less considerable.

The registration districts are divided into *sub-districts*, and each sub-district is made up of one or more civil parishes. The *civil parish* is the smallest unit for Poor Law administrative purposes, but is not used for registration.

The statistics relating to the registration counties are summarized for some purposes in eleven Divisions, viz.: London, South-Eastern, South Midland, Eastern, South-Western, West Midland, North Midland, North-Western, Yorkshire, Northern, and Welsh.

Administrative Counties. These date from the Local Government Act of 1888, which established County Councils. Several of the old counties were divided for this purpose into

* Summary Tables of the Census, Table II.

two or more administrative counties (*e.g.* The Parts of Holland, of Kesteven and of Lindsey in Lincolnshire, East and West Sussex), so that there are now 50 altogether in England and, as before, 12 in Wales. Boroughs which contained over 50,000 persons in 1881, and a few others which had before enjoyed some independence, were left outside the administrative counties and called *County Boroughs;* other boroughs which have since 1881 successfully claimed the possession of a population of 50,000 have been raised to the same rank. There were 75 county boroughs in England and Wales in 1911. Many minor adjustments of county boundaries were made, but, except for the separation of London, the administrative counties (when the subdivision, as in Sussex, is ignored), together with the county boroughs they surround, are nearly co-extensive with the Ancient Counties.

Each administrative county (except London) is divided into *Urban* and *Rural Districts.* The urban districts are either boroughs or simply urban districts.* *Boroughs* are cities or towns which have been incorporated; each has a city or town council consisting of the Mayor, the Aldermen and the Councillors, whereas each other urban district has an urban district council with chairman and councillors. Most independent towns of considerable size or of ancient origin are incorporated. In the Census Reports boroughs are distinguished as C.B. (county borough) or M.B. (municipal borough), but strictly the latter includes the former. Other urban districts are those regions which have been constituted as such, because of their density of population or of their urban character, from time to time by the Local Government Board; they have special powers of administration, chiefly for sanitary and engineering purposes; the most populous of

* The county boroughs are sometimes classified with, sometimes apart from, urban districts. Also they are sometimes included in and sometimes excluded from administrative counties in summary statistics. The County Borough of York stands partly in each of the three Ridings. Great care is necessary in reading the headings of tables on these accounts.

them are on the growing outskirts of boroughs in which their destiny is to be included, others are mining or scattered manufacturing districts.

The boroughs and other urban districts having been subtracted from the county, the remainder consists of *rural districts*, each of which possesses a rural district council. Each urban and each rural district consists of a *civil parish* or group of civil parishes; the parishes in the rural districts have some powers of self-government exercised through the parish councils.

Civil parishes are thus grouped together in one way to make urban and rural districts and in another to make registration sub-districts. An urban district is in general part of a registration sub-district; a rural district in general the remainder of a registration district when the urban districts, if any, are subtracted, the main exceptions being when the registration district is divided by the boundary of an administrative county.

London, for which the administrative and registration counties coincide, is under special laws; it consists of the City of London (with its Lord Mayor) and the City of Westminster and 27 *Metropolitan Boroughs* (each with a Mayor).

For most practical purposes the administrative counties and county boroughs have superseded the Ancient Counties. Birthplaces, however, are supposed to be recorded for the census according to the latter.

The boundaries of civil parishes have been adjusted for this grouping into districts. *Ecclesiastical parishes* may either coincide with ancient or with new civil parishes, or they have been formed by subdividing former parishes, or by carving out a new parish when the population required it.

The division into parliamentary *constituencies* does not necessarily coincide with any of the divisions already named.

Note to ¶ 7, p. 91. There are also official returns of the numbers of seamen employed in vessels of the United Kingdom, and of the number of Railway Servants. See also p. 133 below.

CHAPTER II

VITAL STATISTICS *

1. The most easily accessible source of statistics of births, marriages and deaths is the Registrar-General's Annual Report.† The extracts from it in the Statistical Abstract are insufficient for most purposes. The sources of the Registrar-General's statistics are the familiar marriage and death certificates and register of births, filled in by those responsible on these important occasions. The registration districts have been the same as those used in the population census. [See Note, p. 110.]

Birth- and death-rates are obtained by multiplying the number of births and deaths recorded in a year in a district, great or small, by 1,000 and dividing by the estimated population of the district; the resulting rates are generally given to one place of decimals (thus: 15·3 per 1,000), and in the last chapter it was seen that the population of the whole of England and Wales, at any rate, could be estimated with sufficient accuracy. The marriage-rate is obtained by multiplying the number of marriages by two to get the number of persons and proceeding as before.

2. The following table shows how these birth- and death-rates have fallen in recent years in England and Wales. Similar phenomena are observed in most civilized countries.

* Readers who desire more than this very slight summary should consult *Vital Statistics*, by Dr. Newsholme, Medical Officer of the Local Government Board. See also Dr. Newsholme's and Dr. Dudfield's papers in the *Statistical Journal*, 1905, 1906, and 1908, and Bertillon's *Cours elémentaire de Statistique Administrative*, Ch. VII, XIII, and XXVI–XXXII.

† There are also weekly and quarterly reports and an annual summary for London and large towns; and a Decennial Supplement (of which the last was published in August 1908), giving comparative statistics and much detailed information in Part I, and the relation of deaths to occupations in Part II. The reports of local Medical Officers of Health for districts throughout the country may be consulted with advantage.

ENGLAND AND WALES
Rates per 1,000 of the Population.

	Births.	Deaths.	Marriages.
1871–75 Annual average	35·5	22·0	17·1
76–80 " "	35·4	20·8	15·3
81–85 " "	33·5	19·4	15·1
86–90 " "	31·4	18·9	14·7
91–95 " "	30·5	18·7	15·2
96–00 " "	29·3	17·7	16·1
1901–05 " "	28·2	16·1	15·6
1906–10 " "	26·3	14·7	15·3
1911 " "	24·4	14·6	15·2
1912 " "	23·8	13·3	15·5
1913 " "	23·9	13·7	15·5

It is believed that births are adequately registered, but the possibility should be borne in mind that the regulations recently put in force for the immediate notification of a birth may bring the registration more up to date.

3. If the constitution by age and sex, as shown on p. 98, was the same for all countries and districts, or even constant for the same place over a period of years, we could use the rates here given for comparative purposes. Actually there has been a considerable change in England and Wales in the last forty years; indeed, the falls in the birth- and death-rates are almost * certain to increase the average age of the population.

AGE COMPOSITION OF THE POPULATION OF ENGLAND AND WALES, PER 1,000.

	1871.		1901.		1911.	
	Males.	Females.	Males.	Females.	Males.	Females.
Under 5 years	68	68	57	57	54	53
5–20 "	161	161	154	156	146	146
20–45 "	166	182	181	199	185	203
45–65 "	71	76	71	78	77	84
65– "	22	26	20	26	22	30
	487†	513	484†	516	484	516
M. and F. 20–45	348		380		338	

* If the death-rate had been reduced only for infants, the two falls might compensate each other.

† Here and in many other tables both items and totals are stated in round numbers, and the sum of the items as given is not necessarily equal to the total.

It is clear that so great a change in the relative numbers of the young, the old, and those in the prime of life, must affect the birth- and marriage-rates if these are simply based on the whole population. The problem is to get a true average (see p. 68) in which the number of marriages is referred to the number of marriageable people, and the number of births to the number of married people (with reference also to age *), and to standardize a means of comparison with the same purpose as that used for correcting the death-rate discussed below. Readers interested in this extremely important problem should devote careful attention to the articles in the *Statistical Journal* referred to on p. 102.

4. The general death-rate is also greatly affected by such differences, for the rates differ greatly according to age and sex.

DEATH-RATES AT VARIOUS AGES, ENGLAND AND WALES

	Average 1891–95.		Average 1901–05.	
Ages.	Males.	Females.	Males.	Females.
0– 5	62·9	52·8	53·8	44·9
5–10	4·5	4·6	3·6	3·7
10–15	2·6	2·8	2·1	2·2
15–20	4·0	4·0	3·2	3·0
20–25	5·2	4·8	4·3	3·6
25–35	7·1	6·6	5·9	5·0
35–45	12·0	10·2	9·9	8·2
45–55	19·6	15·2	17·3	13·3
55–65	35·9	29·5	32·9	25·7
65–75	72·5	63·1	67·2	56·4
75–85	149·3	134·4	137·4	121·5
85–	291·6	264·2	283·0	261·3
All ages	20·0	19·8	17·1	15·0

The rate is reckoned for each sex-age-group as the number of deaths in a year per 1,000 of the estimated number living in the group at the middle of the year. The death-rate is greatest immediately after birth. *Infant mortality* is always reckoned as the number of infants under one year old who

* See *Seventieth Annual Report of the Registrar-General*, p. xxv, for comparison of the birth-rates for thirty years on two methods.

die in a given year per 1,000 born alive in that year, and differs from the death-rate as just defined. The following table shows that this mortality is very high, but that it has fallen in the most recent years.

INFANT MORTALITY

England and Wales—

1871–80	1881–90	1891–95	1896–1900	1901	1902	1903	1904	1905	1906	1907	1908
149	142	151	156	151	133	132	145	128	132	118	120

1909	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919
109	105	130	95	108	105	110	91	96	97	89

A population containing an excess of persons between the ages of 5 and 45, and few infants or old people, will show a smaller general death-rate than a normal population if the rates at all the different age groups are the same in both populations.

5. The method of correcting or adjusting the death-rates for comparison of two dissimilar populations is important, and, though complicated in idea, is easy to apply.

The method generally used *e.g.* in the Annual Summary of the *Births, etc., in London and other Large Towns* is as follows: The death-rates for all the towns concerned are first calculated on the hypothesis that in each town the death-rates are equal to those of England and Wales (say in 1891–1900), age group for age group; the differences so found between the general rates from town to town are due solely to the differences of relative numbers at various ages. The

YEAR 1901

Districts or Towns.	Standard death-rate.	Correction factor.	Recorded death-rate 1901.	Corrected death-rate 1901.
England and Wales .	19·15	1·	16·90	16·90
33 large towns . .	17·72	1·0806	18·59	20·09
Rest of England and Wales	19·47	·9835	15·99	15·73
London	17·97	1·0656	17·63	18·79
Plymouth	19·70	·9720	17·89	17·39
Oldham	16·72	1·1453	19·64	22·49
Manchester . .	16·90	1·1331	22·10	25·04

Notice that the correction changes the order of Plymouth relative to London.

resulting numbers are called the "standard death-rates" for the towns. Thus, if London has fewer children and old people than the country in general, its standard is low; if Plymouth has many children its standard is high. The standards given in the table are based on the statistics of 1891–1900.

The quotient obtained by dividing the standard (19·15) for England and Wales by the standard for the district is called the correction factor, and is given in the table for six districts. This is calculated only now and again, and then applied to the figures that are recorded year by year as they come.

In the third column the recorded (or crude) death-rates are given for some districts taken at random.

The recorded death-rate multiplied by the correction factor is called the "corrected death-rate."

Example.—London compared with England and Wales. Correction factor $= \frac{19{\cdot}15}{17{\cdot}97} = 1{\cdot}0656$. Corrected death-rate = (recorded death-rate) $17{\cdot}63 \times 1{\cdot}0656 = 18{\cdot}79$. Hence we see that corrected death-rate = recorded death-rate × $\frac{\text{general standard}}{\text{standard for town}}$. If the town standard is low the process raises the recorded rate in the proportion that it should be raised to counteract the effect of age and sex distribution, as shown by the standard rates.

This is not the most obvious process by which a correction could be made, but it proves to be the easiest to compute, and it can be shown by the principles of averages to yield approximately the same result as the following apparently simpler method: Calculate the death-rates for each age group in the town; apply these to a standard population group by group; the result is the corrected rate, freed from the accident of age distribution. This method would, however, need the estimation of the age distribution every time, and some dozens of arithmetic processes for each town instead of one;*

* If $s_1, s_2 \ldots$ are the numbers per 1000 in the age groups in England and Wales, and $t_1, t_2 \ldots$ in the groups in an assigned town in the standard period, and if $D_1, D_2 \ldots$ are the corresponding standard death-rates,

it is used, nevertheless, in the Annual Reports and given for all the registration counties.

6. The importance to public officials of the study of comparative death-rates can hardly be over-estimated. If the death-rate in a district is above that in similar districts there is *a priori* something wrong, and very careful analysis is needed to determine what it is. Death-rates depend not only on age and sex, whose effect can be tested as in the previous paragraph, but on occupation, as to which statistics are given once in ten years by the Registrar-General, and on occupation combined with age; death-rates are, of course, influenced also by epidemics and by catastrophes, and the years affected in this way must be ruled out of comparison. The most important subject for study at the present time is infantile mortality, which may be regarded as of such a distinct character from general mortality that the latter should be restricted to the rate per persons over five years. There is no doubt that a great part of infantile mortality can be avoided; in considering its magnitude attention should be directed to the age (in weeks and months) of the infant, to the economic position of the parents, to the cause of death, with special reference to obviously avoidable causes and to the annual epidemic of summer diarrhœa, and to the effect on the rate of the presence in the district of workhouses, hospitals and other institutions, where the presence of specially feeble infants may in some cases be expected.

7. Problems relating to sickness and mortality naturally come within the province of medical officers of health, and in many districts these officers present admirable annual reports,

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and $d_1, d_2 \ldots$ the recorded death-rates for the town in any year, and if $\Sigma s = 1000 = \Sigma t$, then $\frac{\Sigma s\text{D}}{1000}, \frac{\Sigma t\text{D}}{1000}$ are the standard rates for England and Wales and the town, $\frac{\Sigma s\text{D}}{\Sigma t\text{D}}$ the correcting factor, $\frac{\Sigma td}{1000}$ the recorded death-rate, and $\frac{\Sigma td}{1000} \times \frac{\Sigma s\text{D}}{\Sigma t\text{D}}$ the corrected rate.

The other method would be to take $\frac{\Sigma sd}{1000}$ as the corrected rate.

tackling the questions of most importance in their localities with statistical and professional skill. It will perhaps be useful to indicate the application of the methods sketched in Part I above to this class of problems.

The most important method is that of *averages* in the form of rates. Besides death-rates, etc., we have the "morbidity-rate" (or "attack-rate") which is the number of cases of a particular disease (multiplied by 1,000 or some other round number) divided by the population, and the "case fatality" rate, which is the number of deaths due to a disease divided by the number of cases. Here, as with death- and birth-rates, the denominator must be chosen carefully; for the morbidity-rate the persons should be grouped by ages, districts, etc., so that the classes with different degrees of liability to the particular disease shall be considered separately. For the "case fatality" rate, great care must be taken to include all the cases, and to be certain of the diagnosis. If differences of treatment (hospital or home) or the efficacy of protection (vaccination, isolation, etc.) are in question, there is always the risk that the ages or economic conditions of the classes considered may differ, and the groups must be made similar before comparison is attempted.

All through vital statistics there is great risk of inadequacy of, and even of mistakes in, *definition*. These arise (i) from intrinsic difficulty of classification and incomplete standardization of description; (ii) from unconscious personal bias of the practitioner; (iii) from the presence of two diseases together, or a disease and an accident; (iv) from the desire to avoid the statement of the existence of certain classes of disease (*e. g.* alcoholism). The presence of any of these may affect the apparent death-rate from any cause, and also the morbidity and case-fatality rate.

In considering questions of cause and effect, liability of various classes, and results of different treatments, the essential thing is to get the exact difference to be considered clearly stated, and then to proceed to analysis by *tabulation*. If the headings of the table prove to be clear and distinct

and to follow the differences needed in the problem, the table is good and relevant. Tabulation, when it is not analysis should either be omitted to save space if quite unimportant, or relegated to an appendix if the data may be wanted at some other time, or fitted into standardized tables in a statistical section if they are needed for comparison. The main line of argument or of information should not be interrupted by tables which do not give definite answers to definite questions.

Accuracy.—There is very great risk in most vital statistics of spurious accuracy. In the practical question as to whether one rate is greater than another, after the classes concerned have been made similar there remains natural variation; if all known circumstances were the same, differences would still be found. All records of births, deaths, marriages, sickness, must be regarded as *samples;* the greater the number of persons considered the more accurate the average obtained from the sample. The only non-mathematical test of this accuracy is the test of subdivision (see Chapter VII above), that is, the finding the amount of agreement if smaller groups are taken; the mathematical tests are extremely important, but should only be used when thoroughly comprehended, and are therefore not summarized here. A very great number of differences that are remarked on, prove on mathematical examination to be only the result of chance variation, and to be no more remarkable than (say) the throwing of double-six twice in succession. Here we can only recommend extreme caution in drawing conclusions.*

As a simple and obvious rule, based on the elementary ideas of accuracy (Chapter II, above), the rate should never be reckoned to more digits than there are in the numerator (number of cases, etc.).

Diagrams should be used sparingly and with reference to the methods discussed in Chapter V above; they are often

* There is much to be said for the establishment of a medical-mathematical office, to which statistical data could be sent for examination, just as bacteriological tests are made.

specially useful in tracing the course of an epidemic, and in the relation of the seasons to the incidence of some diseases. (See *Studies in Statistics*, Dr. Longstaff.)

8. It is often remarked, and has great theoretic and practical interest, that averages arising from apparently quite fortuitous causes are nearly unchanged from date to date. The death-rate attributed to "varicose veins" in England and Wales was between 2·1 and 3·7 per *million* persons living every year from 1875 to 1894; similarly the annual rate for "accident or negligence" was in the same period 703, 662, 632, 667, 602, 589, 608, 583, 592, 567, 549, 540, 558, 528, 528, 565, 574, 553, 576, 537, a series of small variation with a downward trend. It is this partial constancy in the total of events based on very large numbers which makes insurance possible. In these instances the events are nearly independent of each other; as a contrast notice the death-rates when the events are not independent, owing to infection, or to fashion in diagnosis; *e. g.* Influenza, 1875 to 1894: 19, 8, 8, 8, 10, 7, 4, 3, 4, 3, 5, 3, 3, 3, 2, 157, 574, 534, 325, 220.

It is when we obtain approximate constancy or a trend with small variation over a series of observations, as in the case of the general birth-, death- and marriage-rates, and the distribution by sex and by age, that we can apply statistical methods for the elucidation of problems and the tracing of cause and effect.

NOTE.—Administrative were substituted for registration areas in the Registrar-General's Annual Report for 1911.

CHAPTER III

TRADE AND TRANSPORT

1. THE statistics of the External Trade of the United Kingdom are published as follows :—

Early in every *month* a cheap unbound account is issued stating the quantity and value of the exports and imports of each commodity, showing the principal sources and destinations of each, with figures totalled for the months of the current year, and comparative statistics for the two previous years. Home produce is separated from foreign and colonial. Accounts of the movement of bullion and of shipping are also included. The details in this monthly issue are subject to correction.

A bulky *annual* report is issued volume by volume. For 1908, the first volume was issued in June 1909 and contained nearly the same headings as the monthly issues, without shipping, and with comparisons for five years; the second volume followed in August and summarized the trade with each country, giving details for each commodity. A supplementary volume is issued later in the year, distinguishing the apparent and real destinations and sources of trade (see p. 122 below). A separate report is issued for shipping (in August in 1909).

The Statistical Abstract for the United Kingdom, issued in August, summarizes all the statistics of trade and gives complete detail as to commodities, but does not show commodities in relation to countries, for which volume ii of the Annual Report and the Supplement are the only sources.

2. The basis of these returns is as follows: The exporter of goods or his agent is bound to send a statement of the

quantity and value of the goods he is exporting to the proper customs officer, who in general accepts the statement; but every bale, etc., on board ship has to be accounted for before the ship is "cleared," *i.e.* permitted to leave the port.

All imports have to be passed through a custom-house; the importer or his agent hands a statement of the goods he desires to have passed, and the customs officers examine the goods with sufficient care to assess duty, if any, or to verify the absence of dutiable goods. These officials check the values, from current price lists or otherwise, and insist on the furnishing of the requisite details. Returns of the values of imports and exports are further checked at the Central Customs Statistical Office, and inquiry is made if the entries appear unusual or are incomplete.

In this process there is a good deal of room for inaccuracy in detail, which may be important for special classes of goods; but there seems no reason to doubt that the descriptions and quantities are stated on the whole with fair accuracy. The values are often a matter of estimate (*e.g.* in the case of goods exported for sale by a foreign agent), and our only security for accuracy is that in a composite total (see p. 29 above) errors which are not biassed tend to neutralize one another, and that, though there are inducements in some cases to exaggerate value, there are inducements in other cases to under-value.

In the case of exports the value is intended to be that of the goods after all internal transport and dock expenses are paid, that is the value at which the goods are delivered free-on-board (f. o. b.). For imports the value is intended to be that of the goods before they are landed, and includes their cost, insurance and freight (c. i. f.). Thus exports are valued at the moment they pass out of the hands of British shore-labour, and imports before they are handled or pay duty. If the exchange were simply across a land frontier, and the goods of one community were exchanged as a whole against the

goods of another, it is clear that the method described would give equal values for imports and exports.

As a matter of fact goods are often quoted at prices to include delivery; in these cases the value has to be corrected for the trade statistics.

3. The following table gives the total trade statistics for 1907.

UNITED KINGDOM

	£ 000's.		£ 000's.
Imports of Merchandise . . .	645,808 A	Exports of Produce of the United Kingdom . . .	426,035 B
Imports of Bullion .	73,072	Exports of Foreign and Colonial Produce .	91,942
Total Imports	£718,880		
		Total Exports of Merchandise . .	517,977 C
		Exports of Bullion .	67,787
		Total Exports	£585,764

Transhipments under bond £18,824 G.

The total A is always quoted as the value of imports, and B is generally quoted as that of exports.

Goods landed may be transhipped either at the same or another port under bond, that is, without passing out of the control of the customs officials, in which case they are entered as "Transhipment" (G) and not included in imports and exports. Goods which pass out of control of the customs are either for use or consumption in the United Kingdom, or for sale again in another country; all such are counted as imports, but when imported goods come to be re-exported they are declared as of foreign or colonial origin. The value A of imports is then thus composed—

MERCHANDISE ONLY

	000's.
Imports for consumption	£553,866 D
Imports for re-exportation . . .	91,942 E
Total	£645,808

Since the goods are valued afresh for exportation, they are presumably increased in value by the expense of handling them in the country, and the value E is thus a little too great. "A" should be compared with C, and D with B.

Actually no theoretic line can be drawn between goods which are (i) simply transhipped, (ii) goods which are re-exported unchanged, (iii) goods which are done up with new labels and re-exported, (iv) goods which form some constituent part of a machine which is exported, (v) yarn which is exported when woven, (vi) wool which is spun and woven and then exported. (i) is included in neither exports or imports, (ii) is included in imports and in exports of foreign produce, (iii) to (vi) are included in imports and in exports of produce of the United Kingdom. It is not possible to correct this method, but it is important to understand it and consider it in the light of pp. 64–5 above.

Notice that bullion and specie, *i. e.* metallic and coined gold and silver, enter nearly equally on both sides of the account.

No special record is kept of trade from one port to another of the United Kingdom or of islands in the British Seas, except that from 1904, the Department of Agriculture &c. for Ireland has estimated Irish Imports and Exports.

4. Other countries have different methods of definition, valuation and classification.* Before using their statistics, it must be ascertained how imports for consumption, for re-exportation with or without alteration, and exports of national and foreign produce are treated, whether bullion and specie are included, exactly what districts are included in the country concerned, and whether there are any peculiarities in the method of valuation.

A general method is as follows: Goods are valued with the intention of producing results on the basis described above for the United Kingdom. Bullion and specie are

* See Reports of the Committee of the British Association on "The Accuracy and Comparability of British and Foreign Statistics of International Trade," 1904 and 1905.

excluded. All goods entering and leaving the country are included in totals of *General* Imports and Exports; goods for consumption or use in the country and exports of goods which have been produced or undergone any process of manufacture in the country are included in totals of *Special* Imports and Exports. General exports are thus greater than special exports by the value of goods passed in and out of or through the country, and similarly with imports; the differences for exports and for imports are approximately equal.

For the United Kingdom we should have

		000's
General exports . .	C + G	£536,801
Special exports . .	B	£426,035
General imports . .	A + G	£664,632
Special imports (approx.)	D	£553,866

It should be noted that the United States and Canada value imports, not on arrival at the port of destination, as is general, but at the place of manufacture.

5. If we regard the international trade of the world as a whole, a consignment forming part of the special exports of one country may appear under general imports and exports of all the countries it passes through, but will finish as a part of the special imports of some one country. The same consignment will be worth more as imports than it was as exports by the cost of transport (including freight, insurance, transhipment and commissions). The following table shows the relation of the special imports and exports of the principal trading countries of the world for the year 1912. The numbers given are subject to many minute corrections. It is thus seen that imports on the whole are worth about 13% more than exports as a whole. The difference, rather over £450,000,000, is received by those engaged in any capacity in international transport, and of this a very large share appertains to the citizens of the United Kingdom.

INTERNATIONAL TRADE. [Merchandise only, except when noted.]
1912

	Special Imports. 000000's.	Special Exports.	Excess of Imports. 000000's.	Excess of Exports.
Russia and Siberia	£124	£160	—	36
Finland	19	14	5	—
Norway	29	21	8	—
Sweden	43	42	1	—
Denmark	41	33	8	—
Germany	526	440	86	—
Holland	297	258	39	—
Belgium	198	158	40	—
France	329	269	60	—
Switzerland *	82	56	26	—
Portugal	17	8	9	—
Spain	42	42	—	—
Italy	148	95	53	—
Austria-Hungary	148	114	34	—
Greece *	6	6	—	—
Bulgaria † ‖	8	7	1	—
Servia	4	3	1	—
Roumania † ‖	23	28	—	5
Total Continent of Europe and Siberia	2,084	1,754	330	—
United Kingdom	633 ‡	487	146	—
British India	118	170	—	52
Australia	77	86	—	9
British South Africa	42	25	17	—
Canada	141	78	63	—
Other Colonies	123	91	32	—
Total British Empire	1,134	937	197	—
U.S.A.	342	452	—	110
Mexico †	19	30	—	11
Costa Rica †	2	2	—	—
Brazil †	63	75	—	12
Peru	5	9	—	4
Chile *	25	28	—	3
Uruguay	9	9	—	—
Argentina *	77	96	—	19
Total America, other than Canada	542	701	—	159
China	72	57	15	—
Japan	65	55	10	—
Grand Total	3,897	3,504	393	—
Add 13% on U.S.A. and Canada Imports §				
Total	3,960	3,504	456	—

* Including bullion and specie.

† In these countries general imports and exports are given, but the difference between general and special must be small. ‡ Imports less re-exports.

§ In order to make their value comparable with those of other countries which value imports on arrival, while U.S.A. values them at place of manufacture. 13 is the value of x given by the equation $3504\left(1+\frac{x}{100}\right) = 3897 + \frac{483x}{100}$.

‖ Figures for 1912 not available; those given are for 1911.

6. Imports are either received as capital loans, or as payment of interest, or in payment of shipping services, or in payment of exports, or in return for bullion and specie. Except in the case of the gold-producing countries the balance of bullion and specie is in general small. When a country has lent capital (in the form of goods not immediately paid for), interest will return to her year by year in the form of goods. Imports are increased relatively to exports when interest is received or capital called in; exports are increased relatively to imports when capital is sent out or interest is paid. Put otherwise, an excess of imports is due to the earnings of shipping together with the excess of interest over new outputs of capital.* Interest or capital which appears as exports for one country appears also as imports for another.

No such balance can be struck between the trade of any two countries; for example, goods sold by Australia to Germany may be paid for by goods sold by Germany to England, and other goods sold by England to Australia, to take a simple instance; it is only for the totality of external trade that a balance is even conceivable.

7. The table on page 118 shows in some detail the values of imports and exports since 1855; the statistics of imports prior to 1855 were not computed on the same basis. To follow the history of the external trade as a whole, smoothed diagrams (the averages being taken over eight or more years), should be constructed as on pp. 40–2 above. It will then be seen that there has been a general but not uniform upward trend throughout the period, concealed or accentuated by considerable fluctuations.

8. The fluctuations both of imports and of exports are principally due to movements of price, and unless we eliminate these we obtain a very imperfect view of the course of trade. The following chapter shows how considerable these movements have been. The method generally used for studying the quantity, or volume, of trade, as distinguished from its

* For the statistics of interest and capital for the United Kingdom see Mr. Paish's paper, *Statistical Journal*, 1909, pp. 465 *sqq*.

EXTERNAL TRADE OF THE UNITED KINGDOM (000000's)

	Imports, less re-exports.		Exports of Home Produce.	
	Declared value.	Estimated value at prices of 1902.	Declared value.	Estimated value at prices of 1902.
1870	£259	£160	£200	£142
1871	271	165	223	150
1872	296	185	256	162
1873	315	195	255	160
1874	312	200	240	160
1875	316	200	223	159
1876	319	223	201	155
1877	341	230	199	159
1878	316	231	193	160
1879	306	234	191	171
1880	348	254	223	194
1881	334	244	234	211
1882	348	258	241	211
1883	361	275	240	217
1884	327	268	233	220
1885	313	272	213	211
1886	294	270	213	222
1887	303	283	222	231
1888	323	294	234	242
1889	361	326	249	251
1890	356	324	263	250
1891	373	339	247	235
1892	359	339	227	227
1893	346	336	218	222
1894	350	364	216	230
1895	357	384	226	249
1896	385	410	240	261
1897	390	415	234	257
1898	410	437	233	256
1899	420	437	264 (255)*	272
1900	460	442	291 (284)	262
1901	454	454	280 (271)	267
1902	463	463	283 (278)	283
1903	473	469	291 (286)	291
1904	481	477	301 (296)	301
1905	487	473	330 (324)	327
1906	523	489	376 (367)	358
1907	554	500	426 (416)	380
1908	513	484	377 (366)	352
1909	533	494	378 (372)	368
1910	574	503	430 (422)	406
1911	577	520	454 (448)	418
1912	633	560	487 (480)	439
1913	659	580	525 (514)	455
Averages.				
1855–1858	£145	—	£117	—
1859–1863	184	—	132	—
1864–1868	232	—	175	—
1869–1873	278	—	225	—
1874–1878	321	£217	211	£159
1879–1883	339	253	226	201
1884–1888	312	277	223	225
1889–1893	359	333	241	237
1894–1898	378	402	230	251
1899–1903	454	453	282 (275)*	275
1904–1908	512	485	362 (354)	344
1909–1913	595	531	455 (447)	417

* The numbers in brackets exclude the value of ships built at home and sold to foreigners, which was not ascertained or included prior to 1899.

value, is as follows: The prices of all goods for which definite quotations can be made are ascertained for a particular year or short period; the quantities of goods exported or imported are then valued in each separate year at these standard prices; it is then assumed that the differences in value shown for the goods which can be priced are typical for all goods. *E. g.* to take an imaginary example—

Value of imports in (say) 1890, as stated in the accounts, *i. e.* at the prices of 1890, £356 (millions). Take 1902 as year of standard price. Suppose that £300 worth of the 1890 imports can be separately valued, and are found to be worth £273 at 1902 prices, the prices in 1890 being higher than those in 1902; then it is assumed that the whole £356 would be reduced in the same ratio, viz. to £356 × $\frac{273}{300}$ = £324, if all could have been valued.

Such a calculation has been carried out year by year by the *Economist* newspaper, goods each year being valued at the prices of the year before. Recently the Board of Trade has issued annually a statement of the values each year since 1900, as they would have been if the 1900 prices had remained unchanged. Prices, especially of coal and steel, were unusually high in 1900, and 1900 is an unfortunate year to have chosen as a standard; exceptional years should always be avoided, as likely to lead to erroneous conclusions. The table above shows the values when 1902 is taken for the standard year.

The method is open to a good deal of criticism in detail, but there is no doubt that it leads to results that are substantially correct, at any rate over short periods.

It is interesting to notice how small are the actual fluctuations in quantity, as indicated by the values at unchanged prices, especially in imports. Consumption of goods and, to a very great extent, production goes on with little change in times of commercial inflation and depression.

The necessity of some such examination is emphasized by the consideration that the rise of 1*d.* per lb. in the price of raw cotton raises the value of imports by about £8,000,000, and since about four-fifths of the cotton manufacture is for

export, the value of exports is also raised by over £6,000,000: these immense changes would take place without any change in quantity or in the work done by British capital and labour.

9. The tabulation of the statistics of the foreign trade has recently been greatly improved, and the new method has been carried back to 1891 in the Statistical Abstract for 1905.*

The table on page 121 shows the summary statistics for the new tabulation.

The complete meaning of the classification can only be seen by looking at the detailed list in the Statistical Abstract; but it may be mentioned that commodities such as yarn and pig-iron, which are the finished product of one process and the raw material of another, are classed as "mainly manufactured."

In the lower part of the table are shown the values for coal, the principal exported raw material, and of the principal groups of manufactures.

In using the table it must be remembered that 1900 and 1907 were years of exceptionally high prices.

10. The original sources of imports and ultimate destinations of exports cannot always be known. If, for example, wool grown in Turkey were spun in Austria, woven in Germany, sent by rail through Holland, manufactured into ready-made clothes in Leeds, and sold in Canada, it would figure in the export and import statistics of many countries, and its value would be due to the co-operation of many nations. Again, if goods are sent from London to Antwerp for sale, they may pass on to Germany, Russia, Austria or Switzerland without the English manufacturer knowing their destination. Till recently imports have been only stated as from the country from which they were last shipped, while exports have been stated as to the country of ultimate destination, as a rule, since 1894.† Thus Switzerland, Bolivia and Rhodesia,‡ which have no seaboard, have had no place in our statistics.

* A more detailed examination of classes of manufactured goods is given in Cd. 2337, Mem. xii, and continued in Cd. 4954, pp. 48 *sqq.*

† Prior to 1894 they were credited to the country to which they were shipped direct.

‡ In these cases exports have been credited to the port of landing.

CLASSIFIED VALUES OF IMPORTS AND EXPORTS OF THE UNITED KINGDOM (000000's)

	Average 1891-97	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914‡
IMPORTS, whether for consumption or export.*																		
Food, Drink and Tobacco	£185	208	210	220	225	224	232	232	232	238	247	244	254	258	264	281	290	298
Raw Materials and Articles mainly unmanufactured	143	148	151	172	167	169	174	182	188	212	242	204	220	261	248	276	282	236
Articles wholly or mainly manufactured	96	112	122	128	127	133	135	135	143	156	155	143	148	157	166	185	194	160
Unclassified	2	3	2	3	3	2	2	2	2	2	2	2	3	3	2	3	3	3
Total	£425	471	485	523	522	528	543	551	565	608	646	593	625	678	680	745	769	697
EXPORTS of home produce.																		
Food, Drink and Tobacco	£11	12	13	14	16	17	16	17	19	21	22	22	23	26	29	33	33	27
Raw Materials and Articles mainly unmanufactured	22	23	29	44	36	36	36	36	36	43	55	52	51	53	54	59	70	57
Articles wholly or mainly manufactured	196	195	219†	229	224	227	235	244	270	306	342	297	297	343	362	385	411	338
Unclassified	2	3	3	4	4	4	4	4	5	6	6	6	7	8	9	10	11	8
Total	£230	233	264	291	280	283	291	301	330	376	426	377	378	430	454	487	525	430
SELECTED EXPORTS.																		
Coal and Fuel	£16	18	23	39	30	28	27	27	26	32	42	42	37	38	38	43	54	42
Iron and Steel Manufactures	22	22	28	32	25	29	30	28	32	40	47	37	38	43	44	49	54	42
Machinery	15	18	20	20	18	19	20	21	23	27	32	31	28	29	31	33	37	31
Ships †	—	—	9	9	9	6	4	4	5	9	10	11	6	9	6	7	11	7
Cotton Manufactures	66	65	68	70	74	72	74	84	92	100	110	95	93	106	120	122	127	103
Wool ,,	25	23	24	24	22	23	25	27	30	32	34	28	31	38	37	38	38	32

* Excluding transhipment under bond. † Ships were not included as exports prior to 1899.
‡ The statistics for 1914 are not readily comparable with those of previous years.

NOTE.—Since each entry and also each total is given to the nearest £1,000,000, the sum of the items in some cases differ by 1 from the entry for the corresponding total.

German and Russian goods have been entered as imported from Holland, Austrian and Swiss goods from Belgium, and so on. From 1904 a second method has also been used, and now the tabulation on the former plan is relegated to a supplemental volume of the Annual Report.* Importers have stated the country from which goods are actually *consigned* to them; this is generally also the country in which they were produced or manufactured or received their last process of manufacture; exporters have also stated the countries to which goods were *consigned*, which are in general the ultimate destination. The following short table shows the results for certain European countries. The imports in the second column are in a very different proportion from that in the first; the exports generally do not differ in the third significant figure.

Imports 1907.				Exports 1907 (including Foreign and Colonial produce).	
Received direct from		Consigned from	Consignments retained for consumption.	Exported to	Consigned to
	00000's				
Russia	£314	329	306	191	191
Germany	388	572	541	567	567
Holland	368	160	154	190	190
Belgium	283	175	168	194	169
France	528	463	398	335	332
Austria	11	68	64	54	54
Switzerland	0	84	72	0	29
	£1892	1851	1703	1531	1532

A committee of the Board of Trade reported in 1908 in favour of making the principal classifications according to the countries of consignment in the future, and of dropping after a few years that according to the countries of shipment.†

11. Shipping statistics call for little comment except as to

* The Statistical Abstract for 1907 shows the results as in the table here given. That for 1914 gives the countries of shipment in Table 33, and of consignments in Tables 34, 35. From 1915 only consignments are given and the Tables are rearranged. The monthly accounts now give countries of consignment, not of shipment.

† Minutes, Cd. 4346; Report, Cd. 4345.

the meaning of tonnage (see note at the end of the chapter). The Statistical Abstract gives a series of useful and easily intelligible tables on the subject. Every ship is registered as of a definite nationality, which is generally * that of her owners. On *entering* a port of the United Kingdom the ship's papers must be shown, stating whence she came, where she last broke bulk, what cargo she carries, and her registered tonnage. She cannot leave the port till her papers have again been seen, her next destination stated, and the necessary declarations of her cargo are in order. In theory, nothing enters or leaves the United Kingdom without official knowledge.

Steam- and sailing-ships are distinguished. It must be remembered that a steamship carries much more cargo between the same two countries in a year than a sailing-ship of the same carrying power, owing to her greater speed and more frequent journeys. Coastwise and foreign voyages are distinguished; coastwise means between any two ports in the United Kingdom or islands in the British seas; foreign is from or to a port in the United Kingdom to or from a port in a foreign country or one of the British possessions.

None of the tables show the aggregate of the voyages of British ships or any other measure of the work done by them and it must be realized that a great part of the merchant navy carries cargo between distant ports without calling at home at all. The share of our merchant navy in the £450,000,000 shown on p. 116 cannot be estimated accurately.

The Abstract for Foreign Countries shows the aggregate tonnage of the ships engaged in foreign trade under the flag of the principal maritime nations.

Year 1904. Tons 000's

Merchant Navies of	Sailing.	Steam.	Merchant Navies of	Sailing.	Steam.
United Kingdom .	1,803	8,752	Norway	809	643
Germany	579	1,774	U.S.A.†	345	554
Japan	327	797	Italy	570	462
Spain	64	717	Sweden.	266	408
France	653	696			

* It is said that the United States have considerable holdings in ships in the Atlantic trade registered as British.

† In addition U.S.A. had 2,351,000 tons sailing and 3,041,000 tons steam employed in inland and coast navigation.

The measurement of tonnage is not quite uniform, and the line excluding small vessels (whose aggregate tonnage is also small) is drawn differently from country to country.

12. Railway statistics are deficient in the extreme for the United Kingdom. In the Statistical Abstract the principal known facts are summarized. Apart from finance, these are the length of line open (distinguishing single from double or more), the number of passengers, and the weight of minerals and of other merchandise carried. There is no information as to the average or aggregate distance travelled (passenger-miles, and ton-miles), which is used in other countries to measure the work done by railways. The totals are so crude and heterogeneous as to be practically valueless (see p. 65 above), except that over a very few years the total weight carried gives some indication of the upward and downward movements of trade. In the official annual return on railway working (*e.g.* Cd. 4804 for 1908) we have also the totals of "train-miles," distinguishing passenger from goods, that is, the aggregate of the number of miles run by all the trains in a year. This regards as similar a mile run by a crowded suburban train, by a long-distance express, and by a nearly empty local train, and the total is evidently not of general use.

NOTE.—*Shipping tonnage.* The definition and measurement of tonnage are extremely complicated, as may be seen from the Report on the Merchant Shipping Bill (H. of C. 256, 1907), where many examples are given. There are at least four measurements of a ship's size or capacity: displacement, burden, gross tonnage, and net or registered tonnage. The displacement is the weight of the ship (unloaded), which equals the weight of the water displaced; the burden represents its carrying capacity; neither of these is used in the general shipping statistics. The gross tonnage is the number of times 100 cubic feet is contained in the ship, measured according to certain rules; 100 cubic feet is taken as representing the space occupied by a ton of cargo, but a ton of coal occupies only about 45 cubic feet, and a ton

of water about 35 cubic feet; light or loosely packed cargoes occupy more. Net tonnage is obtained from gross by subtracting according to artificial rules space occupied by the engines (with an allowance for bunker and air space), by the crew's and passengers' quarters and the parts necessary for navigation; the remainder (reckoning as before 100 cubic feet to the ton) is supposed to represent the carrying capacity of the ship, and is the registered tonnage. The rules for measurement and deduction differ for different nations, but there has been a wide-spread movement in the direction of adopting the British system. The British rules have been modified from time to time. Actually the registered tonnage does not bear any close relation to carrying capacity, and is extremely artificial. All the recent shipping statistics are given in registered (or net) tonnage; formerly they were given in gross tonnage, but the present method runs back far enough for all practical purposes, and it is easy in comparative statistics to see if there has been a change in this respect. Sailing-ships' tonnage (which has, of course, no allowance for propelling machinery) should be kept distinct from steamships. Marine architects continually try to build so that the registered tonnage shall be as low as possible, since dock dues are charged in proportion to this tonnage, and they take advantage of the rules of measurement so that the deductions allowed shall be as great as possible; in other words, they try to reduce the registered tonnage relatively to the carrying capacity. It follows that the growth of shipping tonnage shown in the tables tends to fall short of the real growth of carrying capacity. Further, the Plimsoll mark, which regulates the weight a ship can carry, was raised in some classes in 1906 to allow a greater weight without altering the registered tonnage. The general result is that the shipping statistics cannot be used for any fine measurements, and are not comparable over a long series of years.

NOTE.—For amount of shipping at a port, and distinction between "arriving" and "entering," see new table, No. 66, in Statistical Abstract for 1910, and introductory notes in Annual Statement of Navigation, 1911.

CHAPTER IV

PRICES

1. PRICES from the ordinary commercial standpoint are of course to be found in the trade journals, and summaries from time to time in the *Economist* and the *Statist*. From the statistical point of view we are only concerned with the change in particular prices over a series of years, and with general price movements. For both purposes the most accessible information is to be found in the tables of the Statistical Abstract, which show the prices of exports, imports, cereals and minerals, and in Mr. Sauerbeck's studies of price movements published annually since 1886 in the March number of the *Statistical Journal*. The Board of Trade Report on Wholesale and Retail Prices (H. of C. 321, 1903) contains a great many records of prices over a long series of years, and interesting charts showing the prices of wheat and of bread since 1800 were published in the *Labour Gazette*, May 1909.

2. The great difficulty in the measurement of prices is in the definition of the commodity to be measured. In the case of the staple raw materials of manufacture, cotton, wool, iron, etc., and the principal raw foods, wheat, sugar, etc., the various grades are to a great extent standardized, and it is only after the lapse of a considerable time that difficulties in exact comparisons are felt; for example, wheat prices can be properly compared over (say) 20 years, but in a century the kind of wheat commonly in use has changed immensely. As the raw materials pass through the various stages of manufacture endless varieties are introduced and the goods continually change their character without changing their name, or qualities which were commonly used fell out of fashion;

for these reasons it is not possible to measure the price of such commodities as cotton yarn or cotton piece-goods over a long period; still less can we reckon the change in price of ready-made clothes, of machinery, of bicycles, etc. Similarly the change in character of live stock, of timber, of everything of which the source varies or which can be modified by man, is readily perceptible after even a few years.

3. The measurement of retail prices is so difficult that neither government departments nor statisticians have as yet made much progress with it. All the varieties of production and all the changes of fashion have their full influence here. It is seldom that goods can be exactly matched, even in external appearance, after a few years; and a more subtle difficulty is present, for the actual quality of goods is very frequently changed with no corresponding change in price, customers demanding articles at the price they are used to and the manufacturer making slight changes in the constituents to preserve his profit. As an illustration of another difficulty, it may be observed that the price of travelling one mile by railway has been nominally 1*d.* since railways existed, but the kind of accommodation and the speed of travelling, have changed completely, and a considerable proportion of the third-class journeys made are in fact charged at a lower rate, We therefore leave the whole problem of retail prices on one side as too complex for the beginner.

4. The Board of Trade prices of imports and exports published in the Statistical Abstract, are obtained by rejecting those commodities, such as pictures, horses, machinery, miscellanea, etc., for which an average price is clearly an absurdity, by dividing the total value of imports or of exports for the year by the total number of units of quantity for each commodity not rejected. The price stated is thus always an average, not a market quotation, and in some cases (*e.g.* carpets and druggets) the divisor is not homogeneous. An apparent change of price is often due to an actual change of quality, and in some cases this change is cumulative, not accidental; for example, if the general run of "heavy broad

	1	2	3	4	5	6	7	8	9	10	11	12
	Wheat.	Refined Sugar	Tea.	Cotton.	Wool.	Jute.	Pig and Puddled Iron	Coal.	Silver.	Index Numbers.		Sauerbeck's Index Numbers.
	per cwt. *s.*	per cwt. *s.*	per lb. *d.*	per cwt. £	per lb. *d.*	per cwt. *s.*	per ton. £	per ton. *s.*	per oz. *d.*	Import.	Export.	
1853–9	12·8	38·2	16·4	3·06	17·3	33·8	3·4	9·3	61·6	155	122	142
1860–4	11·2	34·3	18·4	6·4	16·8	19·6	2·8	9·0	61·3	176	—	146
1865–9	11·8	31·7	19·0	5·7	15·8	17·6	2·9	9·8	60·7	167	—	144
1870–4	12·0	33·8	16·8	3·97	14·3	18·6	4·40	14·4	59·8	160	150	149
1875 .	10·6	30·3	16·7	3·47	15·4	15·1	3·64	13·1	56·7	150	140	133
1876 .	10·4	29·4	16·4	3·02	14·5	14·7	3·12	10·8	52·7	143	130	137
1877 .	12·5	33·8	16·0	2·93	14·4	16·1	2·87	10·0	54·9	148	125	135
1878 .	11·0	29·3	15·3	2·80	13·9	15·3	2·67	9·3	52·6	137	120	125
1879 .	10·6	27·4	14·7	2·76	13·6	13·7	2·57	8·6	51·2	131	112	120
1880 .	11·1	29·2	13·5	2·94	13·7	17·3	3·20	8·8	52·2	137	115	127
1881 .	11·0	28·9	12·8	2·92	13·9	16·1	2·77	8·8	51·7	137	111	122
1882 .	10·7	28·7	12·6	2·93	12·3	14·5	2·82	9·0	51·6	135	114	121
1883 .	9·8	27·2	12·5	2·91	12·1	12·3	2·61	9·2	50·6	131	111	118
1884 .	8·4	20·9	11·8	2·85	12·1	14·1	2·32	9·2	50·7	122	106	110
1885 .	7·8	18·1	12·1	2·86	10·0	11·3	2·18	8·8	48·6	115	101	104
1886 .	7·5	16·7	11·8	2·49	9·1	11·2	2·16	8·3	45·4	109	96	100
1887 .	7·6	15·6	10·6	2·51	10·1	11·2	2·36	8·2	44·6	107	96	98
1888 .	7·7	17·5	11·0	2·59	9·8	12·4	2·13	8·3	42·9	110	97	101
1889 .	7·7	19·7	10·8	2·64	9·8	14·2	2·51	10·0	42·7	111	99	104
1890 .	7·8	16·3	10·6	2·67	10·3	13·3	3·05	12·4	47·7	110	106	104
1891 .	8·9	16·5	10·7	2·59	9·4	12·2	2·63	12·0	45·1	110	105	104
1892 .	7·7	17·1	10·1	2·39	8·7	15·2	2·57	10·9	39·8	106	100	98
1893 .	6·4	18·4	9·7	2·43	8·7	13·2	2·35	9·8	35·6	103	98	98
1894 .	5·3	15·5	9·6	2·06	8·5	13·6	2·30	10·4	28·9	96	94	91
1895 .	5·5	13·3	9·6	1·94	8·1	11·1	2·40	9·2	29·9	93	91	90
1896 .	6·2	13·6	9·5	2·31	8·4	12·2	2·39	8·7	30·7	94	92	88
1897 .	7·4	12·3	9·4	2·09	8·0	11·7	2·41	8·8	27·6	94	91	90
1898 .	8·0	12·3	9·1	1·80	8·2	10·5	2·63	9·8	26·9	94	91	92
1899 .	6·7	12·6	8·8	1·91	8·6	12·7	3·47	10·5	27·4	96	97	98
1900 .	6·8	12·8	8·5	2·61	9·5	14·7	4·20	16·5	28·2	104	111	109
1901 .	6·6	12·2	7·7	2·57	7·5	13·5	3·13	13·7	27·2	100	105	101
1902 .	6·7	10·6	7·2	2·54	7·5	12·8	3·24	12·2	24·1	100	100	100
1903 .	6·8	10·7	7·7	2·80	8·3	13·5	3·15	11·6	24·7	101	100	100
1904 .	7·0	12·3	7·2	3·13	8·7	13·7	2·92	11·0	26·4	101	100	101
1905 .	7·2	14·8	7·2	2·65	9·3	17·0	3·15	10·5	27·8	103	101	104
1906 .	7·0	11·6	7·4	3·09	10·2	22·6	3·50	10·8	30·9	107	105	111
1907 .	7·7	12·0	8·1	3·31	10·3	22·4	3·71	12·6	30·2	111	112	115
1908 .	8·4	13·0	8·0	3·03	9·3	16·6	3·17	12·6	24·4	106	107	105
1909 .	9·3	13·4	8·2	3·09	9·5	15·1	3·24	11·2	23·7	108	103	107
1910 .	8·4	15·6	8·2	4·07	10·2	15·7	3·42	11·6	24·6	114	106	113
1911 .	7·9	15·3	9·0	3·61	10·0	19·9	3·22	11·3	24·6	111	109	115
1912 .	8·5	16·5	8·7	3·20	9·9	21·7	3·69	12·6	28·0	113	112	123
1913 .	8·3	13·4	9·1	3·64	10·3	26·4	4·30	13·8	27·6	113	116	123

Columns 1–6 are the "Average Import Prices" obtained by dividing the values of the commodities imported, as stated in the Statistical Abstract, by the quantities there stated. Columns 7 and 8 are "Export Prices" obtained in a similar way.

woollen tissues, all wool," increased in breadth, perfection of manufacture, and finish, it would still be entered under the same category. The table on p. 128 gives examples of prices (wheat, jute, and pig-iron) where the quality has probably not changed much, of others (cotton and coal) where the relative proportions of different qualities have probably changed perceptibly, but not when few years only are considered,* and of others (tea and sugar) where there has been almost a revolution in the trades. Evidently these prices need interpretation by persons conversant with the industries with which they are connected. Silver, on the other hand, is perfectly defined chemically.

Other prices given in the Statistical Abstract are those of wheat, barley and oats, which are obtained by averaging the records of sales in the various corn markets of the country. The prices of minerals, including pig-iron, at the places of their production, can be obtained approximately by dividing their estimated value by the number of tons produced.

5. *Index-numbers.*—When measurable phenomena (such as prices or wages) are influenced (1) by causes *special* to particular instances, (2) by *general* causes presumably acting on all the phenomena, it is important to disentangle the general causes from the special. Thus the price of wheat is influenced by the weather, acreage under the crop, and the harvests in all the wheat-growing countries; the price of coal by the fluctuations in demand: these are causes special to these commodities. The prices of wheat, coal and all commodities are influenced by the relation of the amount of money and its substitutes to the work that has to be done by them: these are general causes. To determine the effect of the general causes, that is, to determine the general change of price, which varies inversely as the purchasing power of gold, it is necessary to eliminate special causes. This is done by averaging together the price changes shown for a number of different commodities, as follows—

* Even then exceptional years like 1900, when there was a great demand for the coal of South Wales, should be excluded.

As many commodities are taken as possible, for which a perfectly definite price quotation is current, great care being taken to avoid changes of quality; in practice, the number of such commodities is not great, and retail prices must generally be ignored. The average price of a period of years is taken as base, and equated to 100; the prices of other years are then expressed as percentages: *e.g.* from the table above, we should have if we took 1870–79 as base—

	Prices.			Proportionate numbers.		
	Wheat.	Sugar.	Tea.	Wheat.	Sugar.	Tea.
Average 1870–79 .	11·5	31·9	16·3	100	100	100
Year 1890 . . .	7·8	16·3	10·6	68	51	65
„ 1908 . . .	8·4	13·0	8·0	73	41	49

The average of the numbers so found for any year is the index number for that year. This is very nearly the method employed by Mr. Sauerbeck to obtain the index-numbers * given in Column 12.

Another method is to follow the process used in the last chapter (p. 119), thus—Imports in 1890 were valued at £356. At the prices of 1902 they would have been worth £324. Prices in 1890 were therefore higher than in 1902 in the ratio 356 : 324 = 110 : 100. The import index-number for 1890 is therefore 110, when 100 is taken for 1902.

The first of these methods assigns equal importance to each of the commodities chosen, at their average price in the base-period.† To measure the abstract quantity "change of purchasing power" or "appreciation of gold" one commodity is as good as another, and one kind of average is as good as another; it is only necessary to take a sufficient number of commodities to allow the laws of averages free play.

* To facilitate comparison his index for 1902 is equated to 100, and the rest of the numbers raised in proportion.

† A change of base-period may affect the arithmetical importance of special commodities in the average, as may be seen by taking the three commodities used above and taking 1908 as base; but the difference disappears when many commodities are taken, unless abnormal years are deliberately chosen.

The second method is more objective or concrete; it is used to find the value of a definite group of commodities, and this could be done exactly if the data were sufficient. In such cases the method of index-numbers is only a method of abbreviating computation and overcoming the absence of complete information. It is justified, when it can be shown by the principles of averages that the correct objective result must be approximately reached. Similarly, if we wish to find the change over a period in average wages of several groups combined, we could, if we had complete information, work out the actual average year by year; but, in fact, we can only find the ratio changes for the various groups and have to combine these into a wage index-number by the use of suitable weights. In fact, the method suitable for concrete index-numbers differs from that convenient for abstract index-numbers chiefly because weights must be used for the former (unless it can be shown that they would not affect the result), while they can be very often ignored with the latter.

Consideration will, in fact, show that either of these methods is equivalent to comparing weighted averages of the prices. It was stated with illustrations on pp. 18 and 32 above that errors involved in such a process tended to neutralize each other; supposing there to be one ideal true method, all others may be regarded as differing from it by the introduction of many minor errors. Experience shows abundantly that many different methods of computing price index-numbers yield approximately the same result, when proper care is taken to avoid biassed and preponderant errors.

6. Index-numbers may, then, be taken on authority by those who do not desire to follow the extremely interesting analysis on which various methods of obtaining them are based,* as showing with approximate correctness the general change of prices of the group of commodities to which they relate. Sauerbeck's numbers are typical of wholesale prices

* Among many books and articles on the subject, the reader may be referred to Chapter XVIII of *A Plain Guide to Investment and Finance*, by Mr. T. E. Young, in this series.

of raw materials in the United Kingdom, the export numbers of prices of those commodities (principally manufactured goods) which are exported, the import numbers of the great variety of food, raw materials and other commodities imported. If a diagram is made of these three series, their general resemblance will be marked, and it is an interesting exercise to trace the dates and examine the causes of the differences. It is very important to define the group from which the sample prices are selected. It is to be observed that none of those in the table relate to retail prices, nor is rent or the cost of labour involved. They do not represent the inverse* of the value of gold in the hands of the *consumer*.

7. No study of statistical records involving price or value is complete without reference to the general change in the purchasing power of gold thus indicated. The changes may thus be described since 1855: The years 1872–3 were a time of great price inflation, otherwise the general prices fluctuated about the same general level from 1855 to 1874; from 1874 to 1896 prices fell enormously, and but for slight recoveries in 1879–80 and 1887–9, almost continuously, the ratio of prices in 1874 and 1896 being 3 : 2; from 1896 onwards a considerable recovery took place, including two sharp inflations and corresponding falls about 1900 and 1904–8.

8. In dealing with the statistics of Eastern and South American countries, it must be remembered that in many cases statistics are given in silver currency, whose value in terms of gold fluctuates with the price of silver shown in the table, p. 128. The Statistical Abstract for Foreign Countries (published annually by the Board of Trade) contains the information necessary for converting the values.

* If prices fall, the value of gold in terms of commodities rises, and conversely.

CHAPTER V

PRODUCTION

1. The more advanced the stage of manufacture, the further removed from the raw material, the more difficult it is to measure the *quantity* produced by an industry, and the scarcer are the statistics of *value*. We have generally to be content with statistics of the *quantity* of raw material used, and the *value* of that part of the completed goods which are *exported;* for no general record is kept of goods produced for the home market except in the instances given in the next three paragraphs. The Board of Trade has been engaged in working out the results of a Census of Production for 1907, from which we have for the first time general statistics on the subject.*

Part I of the Preliminary Tables was issued in October, 1909; this contains statistics for Coal Mines, Iron and Steel and Tinplate Works and Textile Factories. The following table illustrates the information given—

Cotton Factories of the United Kingdom, 1907.

Output.	Quantity. 000000's	Value.
Yarn	1530 lbs.	£80
Piece-goods	7091 yards	82
Other	—	15
Total		£177
Cost of materials, etc.		£130
Excess of value of output		£47
Number of persons employed—		
Wage earners		560,478
Salaried persons		12,391

* For more detailed account see Appendix III. p. 215.

The intention is to show for each industry the value added to the materials by the processes of that industry. The preliminary tables were completed in nine parts by August, 1911.

2. The Board of Agriculture and Fisheries for Great Britain collects statistics as to the amount of land devoted to various uses, and the estimated yield year by year of the various crops. These are published in the Board's Annual *Agricultural Statistics*, the Department of Agriculture and Technical Instruction of Ireland also publishes an Annual Report, and the results are summarized in considerable detail in the Statistical Abstract. There are no general statistics of the production of wood, fruit, meat or dairy produce, but there are returns as to the numbers of horses, cattle, sheep and pigs. From these latter, combined with expert investigation, estimates have been made* of the quantities of meat and dairy produce produced, imported and consumed. The Board has made detailed inquiries as to the total agricultural product of the country, to be used with the Census of Production (see p. 216). The weight and value of sea fish landed is estimated year by year.

3. The statistics as to minerals are good and complete, owing to the fact that mines have long been subject to inspection; Part III of the General Report on Mines and Quarries† deals with the quantity and the value of the output of coal, copper, lead, tin, zinc and other metals. These figures are summarized in the Statistical Abstract, which also gives the production of "pig-iron" (the first form in which the metal is obtained from the ore, and the raw material of the iron industries and of a great part of the production of steel) both from British and from foreign ores. The amount of these minerals consumed at home can be obtained by adding the imports to, and subtracting the exports from, the home production, in the case of coal and

* *Statistical Journal:* Rew, 1892; Crawford, 1895, 1899; Craigie, 1903; Hooker, 1909; and "Meat and Milk Committee," 1904.

† *E. g.* Cd. 4343 of 1908.

pig-iron; for other metals special knowledge is needed, since both metal and ore are imported.

The production of pig-iron and steel * is estimated as follows—

	Pig-Iron. United Kingdom.	Steel. United Kingdom.
	000's tons.	000's tons.
1891–1895 average	7,040	3,030
1896–1900 "	8,890	4,590
1901–1905 "	8,769	5,125
1906–1910 "	9,765	6,164
1911	9,526	6,565
1912	8,751	6,903
1913	10,260	7,670

4. The number and net tonnage of ships built in the United Kingdom are quoted† in the Statistical Abstract, and distinction is made between ships sold to foreign countries (for mercantile or naval purposes) and those retained under home ownership. (The numbers built for the Royal Navy are not given.) There are great and rapid fluctuations in these totals, and those for single years should never be used in isolation.

5. In the case of the textile trades we can obtain comparative measurements from the raw material, and some further ideas of their progress from the Factory Inspector's statistics of the number of persons, looms and spindles employed. The import statistics of cotton, jute and silk show, when re-exports are subtracted, the amounts of these fibres brought into the home market each year; and trade journals show more accurately the amount actually used. For wool the sources are numerous, and the amounts used are shown in the following extract (see next page) from the annual report of the Bradford Chamber of Commerce.

Of this wool, some is exported after combing, some after spinning, some as cloth or carpets, some as clothes; the remainder is worked up and used at home. Also some

* See H. of C. 284 of 1914.
† See Annual Statements of Navigation, &c.

SUPPLY OF WOOL, ETC. (Million lbs.)

	Foreign and Colonial Wools imported and retained.	Home-grown Wools (total clip less exports).	Wool from imported skins.	Shoddy, etc.	Total quantity of Wool, etc., used.	Estimated value.
						£000,000's.
1880–4 average	235	119	20	123	497	20
1885–9 ,,	301	112	25	101	539	18
1890–4 ,,	343	130	32	118	623	20
1895–9 ,,	405	115	33	132	685	21
1900–4 ,,	380	105	29	145	659	20
1905	371	95	35	180	682	23
1906	412	100	32	190	734	28
1907	489	100	35	210	833	32
1908	425	96	37	180	738	25
1909	453	80	38	205	776	25
1910	503	106	37	226	872	32
1911	525	107	34	210	875	32
1912	514	86	33	201	834	30
1913	537	97	35	205	874	33

foreign yarn is imported. We cannot then say that the total production is proportional to the wool used.*

In the case of wool the value of the output for the home market is probably rather greater than for the foreign. In the case of cotton the value of exports is supposed to be about four-fifths of the total value. Thus a very rough idea can be obtained of the value of the total product, and the value added in manufacture can be estimated by subtracting that of the raw material.

6. For other industries we have only the incomplete indices afforded by the amount of raw material imported, and by the value and quantity of manufactures exported; since in most cases some of the raw material is produced at home and a great quantity of the manufactures are used at home, and since the proportions of home and foreign production and consumption vary, it is generally prudent not to base any conclusions as to production on such imperfect data. In the case of the manufactures which contain iron or steel, however, the very considerable increase in the weight and value exported, as shown in the Statistical Abstract, should be noticed.

* See *Economic Journal*, 1905, pp. 584–590, for statistics as to the growth of the woollen industry.

CHAPTER VI

WAGES

1. STATISTICS of wages are very plentiful; but there are so many different ways of reckoning and paying wages, and such diversity in the methods of stating rates of wages, that these statistics are extremely difficult to handle, and give rise to many misunderstandings.

Wages may be paid by time or by piece. In the former case the rates of wages are so much per hour, per week or other period. The payment does not nominally depend on how much work is done, but there is very often an understanding as to what constitutes an hour's or a day's work, or as to how long a particular job should take. Rates of time wages are very generally agreed on between employers and Trade Unions, and when this is the case there is usually no difficulty in ascertaining them. There is generally also an agreement as to the number of hours which constitute a week's work; the recognized payment for this number of hours is known as the wages for a "normal week," and this payment is the rate generally quoted. It should be observed that in those industries which are carried on at a disadvantage by artificial light the "normal week" is shorter in winter than in summer. Where, as in the building trades, payment is by the hour, there is no certainty that a man will obtain employment for the whole week, and in any case there is loss of time in changing from one job to another in outdoor building work. In this case wage statements generally give the rate per hour and the number of hours which constitute a full week's work season by season. Overtime, that is time outside the scheduled hours that constitute the normal week, is generally paid for at a higher rate. In some trades over-

time is so frequent as to make an important difference in average earnings; in others undertime is common. Besides the week's wage there are in many cases bonuses for regularity or rapidity of work, special rates for special work (*e.g.* harvesting), payments other than money, as when an agricultural labourer has a house at a cheap rate or land to cultivate for himself or perquisites of any kind, or a coal-miner obtains house-coal at a low price. It is thus necessary to have special knowledge of the conditions of employment in each trade before using the bare statements of weekly rates of wages.

2. Piece-rates are, of course, rates of payment for the performance of defined tasks. They very frequently are arranged between employers and employed in the form of elaborate piece-lists (or price-lists, as they are frequently called), which define the exact nature of the task and show innumerable variations of payment corresponding to the various peculiarities of material or machinery by which the work is lightened or made more arduous. The "prices" are usually arranged with a view to the amount of work an ordinary man can do at ordinary pressure in a normal week, so that the week's earnings shall depend rather on the skill or vigour required than on the accident of the special job. Thus printers are paid at a higher rate the smaller the type used. Coal-hewers are paid more per ton when working in narrow seams than where the coal is more easily obtained, Weavers are paid more per yard woven for every additional complexity of the loom. In the large, thoroughly organized industries, especially in the cotton manufacture and in mining, this equalization of earnings is carried to an extraordinary complexity, and the lists are frequently adjusted to suit new conditions as they arise. It is obvious that a piece-list in itself does not give any information as to earnings.

There is in reality no well-defined distinction between payment by piece and payment by time, for time-rates often imply a definite amount of work, and piece-rates are often arranged to produce a definite total of earnings. In fact, there are many methods of payment which are partly on a

time- partly on a piece-basis; for example, in engineering, when time-rates are paid, a definite number of hours is sometimes allotted to a job, these hours are paid for however rapidly it is done, and in addition a bonus paid for rapid work.* The distinction between the methods of payment is, however, of statistical importance, for it is much easier to obtain correct accounts of the week's earnings when on a time-basis, for the statements are given in the form wanted, and there is little variation from man to man; while earnings on piece-rates vary greatly according to skill, opportunity and energy, and information has to be obtained by special inquiries as to individual earnings firm by firm; further, in many occupations on a piece-basis the hours of work vary from man to man and from week to week.

3. Changes of piece-rates are, in the larger industries, at any rate, made by a general percentage increase or decrease of the rates paid to many classes of operatives at once. Thus on June 1, 1909, the rates for about 190,000 coal-miners in South Wales and Monmouthshire were decreased "$7\frac{1}{2}$ per cent., leaving wages $47\frac{1}{2}$ per cent. above the standard of 1879" that is to say, before the change rates were 55 % above those which were arranged in 1879, and have been modified in various details since; the reduction was $7\frac{1}{2}$ % off these 1879 rates, but $7\frac{1}{2}$ on $147\frac{1}{2}$, *i.e.* only 5 % (nearly), on the rates immediately before the change. Where the current rates differ greatly from the standard, it is very important to know on what basis the change is reckoned.

It by no means follows in this case that the ordinary earnings in June 1909 were exactly $47\frac{1}{2}$ % higher than those in 1879. Rates may remain stationary while facilities for production improve, or while the normal week is shortened. In the cotton industry, in particular, slight improvements or alterations of machinery are continually being made, which result in greater productiveness by the operatives, with or without additional intensity of work. Sometimes a nominal

* For illustrations of these methods see *Methods of Industrial Remuneration*, Schloss, 1892.

reduction is exactly counterbalanced, so far as the week's earnings are concerned, by increased ease of production. In all industries where piece-rates are common it is necessary to make occasional inquiries as to actual earnings under ordinary conditions in a normal week, to check the results shown by the percentage changes of rates. These changes do not always show the exact date nor even the approximate amount of the resulting changes in earnings, for the operatives often try to make the same earnings after a reduction as before by more intense application, or neutralize an increase by more leisurely work, and it is only after an interval that the earnings settle to a new level. Making allowance for these tendencies, it remains true that in general earnings for a full week change nearly at the time of the published dates of change of rates, and roughly in proportion to these when they are considerable.

4. In the case of time-rates, the information available is easier to use. The rates generally quoted are those recognized both by the Trade Unions and by the Masters' Associations (if both are effective bodies), and changes are the result of public negotiations. The Trade Union rate is a minimum below which no member of the union is allowed to accept employment; in trades where the union is strong this regulation is actually followed throughout large districts, and even non-union men are unlikely to receive less; where the union is weak the so-called minimum may be a rate which the workmen wish to get recognized, while many are in fact working for less. Special knowledge is, of course, necessary for each trade and district before the actual significance of the rates can be known. It is often supposed that the Trade Union rate is a maximum as well as a minimum; this is not the case in those important industries where there is scope for skill and intelligence; wage-sheets show that payments range several shillings a week above the minimum rate. In fact, it should never be assumed that the Trade Union minimum is the average, or that it bears the same relation to the average over a series of years. As in the case of piece-rates actual

inquiries as to earnings must be made from time to time to correct the impression given by detailed statements of changes.

5. Changes in earnings take place also in many other ways. Where, as in the case of railways or the police, the men are graded and promoted from grade to grade, or receive additional payment in the same grade as their period of service lengthens, a change can be made by an acceleration of promotion or of increase, without any change in the schedule of rates. Where processes of manufacture are changing, it may easily happen that the rates fixed for work at new kinds of machinery result in earnings above or below those made formerly by the operatives who tend them. Such changes are continually taking place in all mechanical industries, and the whole manufacture and the relative numbers at various wage-levels may be revolutionized without a single change of rates taking place. This is only one aspect of a wider process; for in a progressive country some industries are always growing and new industries introduced, while others are stationary or decaying; young persons enter the former trades and find no opening in the latter, and so the population shifts imperceptibly from industry to industry. This tendency results on the whole in an increase in average earnings of the working-class as a group, over and above that shown by changes of earnings in particular industries. Such changes, whether within an industry or in all industries together, can only be measured by occasional complete inquiries as to earnings, combined with estimates of the numbers employed.

6. The official information as to rates of wages is as follows: The Labour Department issues from time to time statements of the time-rates recognized in several industries and in many districts, and also publishes abridgements of price-lists and sliding-scales * in force. The time rates are

* Sliding-scales are arrangements (formerly prevalent and still existing in the coal and iron and steel industries) by which recognized rates change by defined amounts in accordance with the rise or fall of the prices realized for the products of an industry. They have in recent years been superseded in important cases by other methods of adjustment.

summarized in the annual report of the Department. A report "on changes in rates of wages and hours of labour" is published annually, recapitulating and supplementing the details shown monthly in the *Labour Gazette*. More general inquiries (or censuses of wages) were made as to earnings in the years 1886 and 1906; the results of the former were published in a series of volumes from 1889 to 1893, those of the latter are contained in eight reports, of which the seventh was issued in Feb. 1912. There have also been special reports on Agricultural Wages in 1900 and 1905. A great part of what is known officially as to general changes of wages is printed and discussed in the three series of *Memoranda relating to British and Foreign Trade and Industrial Conditions*, generally known as the "Fiscal Blue Books" (Cd. 1761, 2337, 4954). The annual publications are full of important information, but they give no data as to the changes indicated in paragraph 5 on the preceding page; in fact, such changes are ostentatiously ignored. The wage census of 1906 will, it may be hoped, when used in conjunction with occupation statistics (see p. 91 above), make possible a general view of the result of changes of all kinds during 20 years; but it will certainly not be an easy matter to make the comparison.

7. The following are examples of the information as to changes of wages and hours tabulated by the Labour Department:—

Locality.	Occupation.	Date of change in 1908.	No. of work-people affected.	Particulars of changes.
1. Preston	Carpenters and joiners	July 1	360	Advance of ½*d.* per hour (9*d.* to 9½*d.*).
2. North Stafford-shire	Blastfurnace-men	1st making-up day in October	600	Decrease under sliding scale of 3¼%, leaving wages 19¼% above the standard.
3. London	Printers' assistants (rotary machines)	1st pay in January	2,400	Advance of 1*s.* 6*d.* per week.

These would lead to the following items in a summary table :—

Industry.	No. of persons affected.	Net amount of changes per week.
1. Building trades . . .	360	2s.* × 360 = £36.
2. Pig-iron manufacture . .	600	$9\frac{3}{4}$d.† × 600 = £24.
3. Printing, etc. . . .	2,400	1s. 6d. × 2,400 = £180.

* Assuming that 48 hours is the length of the normal week averaged through the seasons. The summer hours are given as $49\frac{1}{2}$.

† Assuming, for the sake of the calculation, that the standard which is not stated is 25s.

The complete summary for 1908 is as follows :—

CHANGES OF RATES OF WAGES IN 1908.

Groups of trades.	No. of workpeople affected.		Net amount of change per week in the wages of those affected.	
	By increases.	By decreases.	Increases.	Decreases.
			£	£
Building trades . .	9,110	3,091	226	—
Coal-mining. . .	100	283,150	—	47,085
Other mining (iron, etc.)	90	14,220	—	2,936
Quarrying . . .	12	2,661	—	290
Pig-iron and iron and steel manufacture .	36	69,247	—	9,656
Engineering and ship-building trades .	19,931	83,531	—	4,050
Other metal trades .	219	1,654	—	207
Textile trades . .	12,026	734	1,132	—
Clothing trades . .	2,112	183	149	—
Printing, etc., trades .	10,808	12	956	—
Glass, brick, etc., trades	5,637	1,864	208	—
Other trades . .	2,703	3,678	—	368
Employees of Public Authorities . .	56,543	191	2,750	
Total . .	119,327	464,216	Net Decrease 59,171	

The numbers are exclusive of 379,790 persons in coal-mining and the iron and steel manufacture, whose wages rose and fell by equal amounts in the year. Individuals are only counted once, however often their wages were changed.

Changes in rates of agricultural labourers, seamen, and railway servants are not included in this table, but are the subject of separate analyses further on in the report.

8. It is noticeable how largely coal-mining wages account for the totals. This has been generally the case since the beginning of these records. Wages in the coal industry and in the manufacture of iron and steel change frequently, depending as they do in many cases on the ascertained selling prices of the products. The wages fluctuate more widely than wages in general, and the changes are in no way typical of changes of average wages in the whole sphere of industry. Unfortunately these changes are the most obvious, and are frequently given too much importance in speeches and writings. The actual rise and fall in these special industries can only be ascertained by observing them over the long period of the ebb and flow of industry. The table on page 145 shows the registered changes from the beginning of the series.

Thus the net increase in the week's wage bill shown by these changes is only £648,000 in 22 years, while the difference between the minimum of 1895 and the maximum of 1900 was only £437,000, and apart from mining, this increase was only £150,000. Now the total of wages other than agriculture in the United Kingdom is probably £15,000,000 per week or more. It is incredible that changes should have been so slight, and it seems certain that the totals of these registered changes cannot be properly applied to the total wages bill.*

The use of these records is of a less general nature; when mining is subtracted, the remainder shows in what years wages were rising and when falling, and to some extent when the movement was rapid and when slow. The more detailed statements relating to separate occupations in separate towns are of the greatest use in making it possible to keep the records of time- and piece-rates up to date.

* In any case, the total wage bill would grow about 1% per annum from the increase of the population, and this would be additive to any increase shown in the tables above.

NET GAIN OR LOSS TO WEEKLY WAGES YEAR BY YEAR.

	Mining and quarrying.		Pig-iron and iron and steel manufacturers.		Textile industries.		Other industries.		Total.	
	Gain. £000's	Loss.	Gain. £000's	Loss.	Gain. £000's	Loss.	Gain. £000's	Loss.	Gain. £000's	Loss
1893	15	—	—	—	—	1	—	1	13	—
1894	—	47	—	1	—	—	3	—	—	45
1895	—	31	—	—	—	—	3	—	—	28
1896	—	5	2	—	—	—	29	—	26	—
1897	7	—	20	—	—	—	4	—	31	—
1898	58	—	3	—	—	—	20	—	81	—
1899	54	—	14	—	6	—	16	—	90	—
1900	173	—	15	—	6	—	15	—	209	—
1901	—	63	—	19	—	—	5	—	—	77
1902	—	73	1	—	—	—	—	—	—	72
1903	—	33	—	1	—	—	—	4	—	38
1904	—	32	—	3	—	—	—	4	—	39
1905	—	14	2	—	10	—	—	—	—	2
1906	28	—	5	—	13	—	11	—	57	—
1907	176	—	7	—	12	—	6	—	201	—
1908	—	50	—	10	1	—	—	—	—	59
1909	—	56	—	2	—	8	—	3	—	69
1910	6	—	2	—	2	—	5	—	15	—
1911	—	10	1	—	1	—	43	—	35	—
1912	80	—	10	—	15	—	34	—	139	—
1913	105	—	2	—	10	—	58	—	175	—
1914	—	28	—	5	1	—	37	—	5	—
Result in 22 years	260	—	43	—	68	—	277	—	648	—

9. The course of wages, whether in separate industries or as a whole, has been studied by the help of these detailed changes together with similar information in earlier times as to Trade Union rates and the changes in piece-rates. These data have been pieced together with the help of evidence and estimates as to actual earnings drawn from a great variety of sources in a series of articles in the *Journal of the Royal Statistical Society* by Mr. G. H. Wood and the present author, of which the first appeared in 1895. All results are tentative till checked by the wage census of 1906, but there is sufficient evidence to support the statements of the following table, as showing the general movements of rates of wages with fair accuracy. It is to be remarked that in the long run wages for work of any particular grade of skill approxi-

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mate to each other, so that a sample which includes the most populous industries must be fairly typical of industries all together.

The working up of the data is actually accomplished by means of index-numbers on a basis generally similar to that of price index-numbers, but the details are more complicated and too technical for discussion here. The principle is to take as data the changes recorded, which can be ascertained, rather than the actual earnings, which can be stated in many different ways according to the bias of the informant. Thus in the table on page 147 the average wage in each industry is taken as 100 in 1880, and the estimated average for other years is given as a percentage of the average in this standard year.*

The column headed "general" shows the course of the average of the wages of all adults employed in all the industries for which the necessary calculations have been made (including the four groups in the following columns), allowing for the shifting from one industry to another and from grade to grade within the industries. The following four columns show similar figures for four important industrial groups. The last column shows the unweighted average (that is, the average of certain rates without reference either to the numerical importance of the different industries, or to the relative growth of some industries) as given in the "Fiscal Blue Books" (Cd. 1761 and 2337), and in the XVIth Abstract of Labour Statistics (Cd. 7131).

The general conclusion from any of these columns is that wages were nearly stationary from 1880 to 1887, rose rapidly from 1887 to 1891, were again stationary till 1897, rose rapidly to 1900, fell very slowly till 1905 back to the level of 1899, rose again in 1906–7, and fell in 1908–9; then a considerable rise took place in 1912 and 1913. Wages at the maximum of 1907 were higher than in 1900, and considerably higher than at any previous date. Wages at the end of 1913 were probably higher than in 1907.

* 1880 is taken simply for convenience of working. The results shown do not depend at all on what year is taken as standard.

INDEX-NUMBERS OF AVERAGE RATES OF WAGES.

Years.	General.	Textiles.	Agriculture.	Building.	Engineering.	Board of Trade Unweighted Average.
1880	**100**	100	100	100	100	*100*
1881	**100**	104	99	100	103	*102*
1882	**103**	104	97	100	105	*103*
1883	**103**	105	96	100	105	*103*
1884	**103**	105	94	100	104	*102*
1885	**101**	104	93	100	103	*101*
1886	**100**	103	91	100	100	*100*
1887	**101**	104	94	101	101	*100*
1888	**104**	108	96	101	104	*102*
1889	**110**	108	97	103	108	*105*
1890	**114**	111	100	104	111	*109*
1891	**115**	113	100	104	111	*110*
1892	**115**	115	100	105	109	*109*
1893	**115**	115	99	107	108	*109*
1894	**115**	115	99	107	108	*108*
1895	**115**	116	97	108	108	*107*
1896	**115**	116	97	109	111	*109*
1897	**116**	116	99	111	113	*110*
1898	**120**	116	101	112	116	*112*
1899	**123**	120	103	113	119	*115*
1900	**130**	123	109	115	119	*120*
1901	**128**	123	110	115	119	*119*
1902	**126**	123	110	115	118	*118*
1903	**125**	123	110	115	117	*117*
1904	**123**	123	110	115	117	*116*
1905	**123**	127	110	115	117	*117*
1906	**126**	127	110	115	119	*119*
1907	**133**	131	110	115	119	*123*
1908	**130**	131	110	115	117	*122*

The Board of Trade's average for subsequent years was, 1908, 122; 1909, 1910 and 1911, 121; 1912, 124; 1913, 128.

On the same basis as that of the first column, the previous maxima and minima for the general average were about—

	Index-number.
1850	68
1855	79
1858	75
1866	90
1868	87
1874	106
1879	99

The numbers in this last table are computed from Mr. G. H. Wood's table, pp. 102–3 of the *Statistical Journal*, 1909.

10. All the statistics of the preceding paragraphs (8 and 9) refer to rates of money wages of persons working full time in a normal week, excluding casual workmen and others not regularly attached to a definite trade. They refer mainly to men, but include the very large numbers of women employed in the textile industries. Two important adjustments must be made before they are applied to measure the economic well-being of the working-class, one for unemployment, the other for the change in the purchasing power of money. The following chapter shows that employment is more regular when wages are rising and *vice versâ*, and that over a long period unemployment in such a group of industries as those considered has neither increased nor diminished perceptibly. The effect of allowing for unemployment would therefore be to increase the fluctuations without affecting the trend of the series shown in the first column of the table on p. 147.

As regards purchasing power in retail commodities, it was stated in Chapter IV above, that the measurement was very difficult; in fact, authorities do not agree as to the movement of prices, especially when rent is included. The following table shows the results of a calculation by the present author.*

	Rates of money wages.	Prices.	"Real" wages.†
1852–1870	Rising fast	Rising	Rising considerably in the whole period
1870–1873	Rising very fast	Rising fast	Rising fast
1873–1879	Falling fast	Falling fast	Nearly stationary
1879–1887	Nearly stationary	Falling	Rising
1887–1892	Rising	Rising & falling	Rising
1892–1897	Nearly stationary	Falling	Rising
1897–1900	Rising fast	Rising	Rising
1900–1910	Fluctuating	Rising	Falling
1910–1913	Rising	Rising	? Stationary

* Adapted from Appendix to *Dictionary of Political Economy*, p. 801.

† For another view see Mr. Wood's article just quoted (*Statistical Journal*, March 1909).

PERCENTAGE OF ALL EMPLOYED, CLASSIFIED BY EARNINGS IN THE LAST WEEK OF SEPTEMBER 1906.

	Under 5s.	5s. and under 10s.	10s. and under 15s.	15s. and under 20s.	20s. and under 25s.	25s. and under 30s.	30s. and under 35s.	35s. and under 40s.	40s. and under 45s.	45s. and under 50s.	50s. and over.	Average.
COTTON—												s. d.
Men	—	—	2·5	14·0	24·1	19·3	12·8	8·7	8·5	5·6	4·5	29 4
Women . . .	·6	5·0	21·5	34·5	28·3	9·0	1·1	—	—	—	—	18 3
Lads and Boys . .	11·6	21·7	40·2	21·1	4·2	1·1	·1	—	—	—	—	11 6
Girls	14·3	37·6	33·3	11·1	3·2	·5	—	—	—	—	—	10 0
WOOLLEN AND WORSTED—												
Men	—	—	5·0	14·7	30·3	20·6	17·6	6·1	2·7	·9	2·1	25 11
Women . . .	8	15·5	53·4	22·3	6·8	1·1	·1	—	—	—	—	13 4
Lads and Boys . .	19·6	43·0	29·7	6·9	·8	—	—	—	—	—	—	8 11
Girls	17·8	54·4	25·9	1·8	·1	—	—	—	—	—	—	8 2
CLOTHING TRADES—												
Men	—	—	6·2	10·4	20·6	24·3	19·3	8·0	5·0	2·0	4·2	28 3
Women . . .	2·0	23·9	43·7	21·4	6·2	1·5	1·3	—	—	—	—	13 0
Lads and Boys . .	10·2	46·8	27·0	13·2	2·5	·3	—	—	—	—	—	9 7
Girls	39·4	51·7	8·0	·8	·1	—	—	—	—	—	—	5 8

The prices included are principally those of food. "Real" wages mean wages expressed in terms of commodities, that is, money wages corrected for change in purchasing power. It is very noticeable that periods of rapid increase of wages have been those also of rising prices, which have neutralized to some extent the benefit of the wage-increase; and that periods of stationary wages have been those of falling prices, which have had practically the same effect as an increase of wages with an unchanged price.

11. All the preceding figures apply to averages, not to individual persons. We have extremely few records of the earnings of individuals for periods longer than a week, though information of a difficult and complex nature is accumulating as to the number of weeks' work and the amount of overtime or lost time obtained or obtainable *in a year* in various occupations, and the variation from year to year. On the other hand, we learn from the Wage Census of 1906,* the relation of the *weekly* wages and earnings of individuals to the average. The table on p. 149 shows in abstract form the kind of information obtained. The earnings are those of all persons, whether working full time, overtime, or short time. The men earning less than 15*s.* were in most cases on short time, and were possibly in a few instances earning money also in other places. The boys and girls earning less than 5*s.* in the cotton and woollen industries were generally half-timers. The statistics refer to the returns obtained from all the principal districts for these industries in the United Kingdom. Tables showing the earnings of those who worked the normal week are also given in the Reports. Information was not obtained for mining. There is still need for a careful comparison of the results of this enquiry with an earlier one for 1886, together with all the other sources of information alluded to in this chapter, before the history of wages 1880–1913 can be completed.

* For industries included see p. 212 below. No general report has been published.

CHAPTER VII

EMPLOYMENT

1. We are entirely dependent on the Labour Department of the Board of Trade for statistics of the amount of employment and unemployment. As in the case of wages there is practically no official information, except as to workpeople regularly attached to a trade; the conditions and numbers of casual workers, of those in the so-called sweated trades, of persons physically or intellectually incompetent to do regular work, of habitual vagrants, and of others chronically unemployed, under-employed, or unemployable, are not the subject of official statistics; in fact, there is practically no evidence, official or statistical or of other kinds, to show the actual numbers in these classes, or to show whether these numbers are stationary, decreasing or increasing.

In several occupations and industries employment (or, at any rate, payment) is practically regular, modified only by occasional overtime and by the temporary engagement of extra hands in busy months. Among these are the Army, the Navy, the government services, such as the Post Office, municipal and other local government services (police, sanitation, etc.), and the railways, and perhaps other land transport services. Nearly as regular as these are domestic service, and the manufacture and distribution of ordinary food and drink. There are thus large sections of the working population who are not directly affected by the question of unemployment.

Next to these come a group of industries, in which certainly more than two million persons are employed, in which it is the custom to regulate the working week in relation to the demand for the product, employing nearly the same number

of persons in good trade and in bad, but working short time when the market becomes overstocked. In these cases the employees suffer from reduced earnings, and may benefit by some enforced leisure, but are not actually unemployed. Coal-mining is the most conspicuous industry of this group. The textile trades (cotton, wool, and others) organize employment with a similar result; short time is worked, or the work is spread out among the operatives, when the demand is slack; but the great number of those employed in moderately busy times draw some wages nearly every week. In times of pressure, however, the textile industries generally find it possible to engage more operatives, whence it appears that there must be a reserve army who are not regularly at work; part of this army consists of married women who have withdrawn from factory work, but return from time to time. Of course these industries gradually grow or diminish quite apart from the fluctations between periods of good and bad trade.

The ordinary statistics of *unemployment* do not relate to either of these large groups. The *Labour Gazette* gives every month tables which indicate the amount of *employment* in industries of the second group.* Thus for mining and quarrying the average number of days worked per week is given; for the cotton, woollen and worsted, linen, jute, lace, hosiery, silk, and boot and shoe industries, and some others, the changes month by month in the number of persons employed and in the aggregate of wages paid, by a very large number of firms, are shown in a series of tables. These tables are not easy to use for the purpose of tracing changes in wages or numbers over a long period, for returns are not obtained from exactly the same firms throughout, and they do not include the whole trade; but they show very clearly when trade is declining or improving, they afford a check on other estimates of wages, and, combined with statistics of output, indicate how various industries meet the ebb and flow of demand.†

* Statistics from the Labour Exchanges are also given.

† For an interesting discussion of this point, and a general analysis of unemployment statistics, see *Unemployment in Lancashire*, Chapman and Hallsworth, 1909.

2. For the manufacture of pig-iron and tinplates, for iron and steel works, and for dock-labour, the *Labour Gazette* shows the number of furnaces or mills in operation, or the number of persons employed. In these industries bad times are not met by shortening the week or spreading out the work among the operatives, and persons are therefore actually unemployed; but there are no distinct statistics of the numbers so unemployed, and estimates would have to be made, if it were possible, by noticing the maximum ever employed, and taking the deficit from this maximum as the number out of work. The statistics are hardly sufficiently complete for this purpose.

There are no definite statistics for agricultural labour or for the ready-made or "bespoke" clothing trades, but verbal accounts are given every month of the state of employment For seamen we have records of the number shipped at the various ports.

3. The only important industrial groups not named in the previous paragraphs are the printing, building, furnishing and woodworking, engineering and shipbuilding trades. The ordinary statistics of unemployment include considerable numbers of these groups.

"LABOUR GAZETTE," October 1909.*

Industries.	Membership of the Unions from whom returns were obtained, September 1909.	Number unemployed at end of September 1909.	Percentage of Membership unemployed.
Building	58,917	6,432	10·9
Coal-mining. . . .	139,746	1,669	1·2
Engineering . . .	171,370	18,592	10·8
Shipbuilding . . .	57,280	12,855	22·4
Other metal trades . .	41,504	2,286	5·5
Textiles	115,821	2,721	2·4
Paper, printing, and bookbinding	59,127	3,820	6·5
Woodworking and furniture	35,165	2,719	7·7
Miscellaneous . . .	16,790	655	3·9
Total . . .	695,720	51,749	7·4

* Supplemented by additional details furnished by the Department.

The numbers included from coal-mining and textiles are an insignificant proportion of the aggregate in these industries, and contribute little to the total of unemployment thus measured.

Among the Trade Unions of the United Kingdom only the minority, who pay allowances to their members when out of work ("unemployed benefit"), keep a record of the members unemployed. Reports are obtained from this minority by the Labour Department of those who are on the unemployed books (whether in benefit or not) of the various branches at the end of each month, together with the membership of these branches. The table just given is compiled directly from these reports. The numbers do not include persons on strike, sick or superannuated, who draw other "benefits" from the unions.

The numbers for the building trades depend only on carpenters and plumbers. The Operative Bricklayers Society has no unemployed benefit except for travelling. In the winter months carpenters, painters and plumbers have more employment than those in other building operations, and the percentage of unemployment for all the building occupations would be higher in the winter than that shown in the returns. There is also much under-employment, or lost time, in the building trades, where the hourly system of engagement is prevalent, which is not shown in this table.

On the other hand, the engineering and shipbuilding and, perhaps, the printing trades are adequately represented.

The figures refer almost exclusively to artisans; labourers' unions do not generally pay unemployed benefit.

These returns are, therefore, merely a sample of the facts of unemployment, and there is little reason for taking the resulting percentage as applicable to industry as a whole. It is sometimes supposed that labourers are more frequently unemployed than artisans; but this is not the case when they are attached to industries in which skilled work is prevalent, for the whole group, men and women, boys and girls, skilled and unskilled, co-operate, and the

labourers cannot stop unless the work is stopped. Agricultural labourers obtain regular work if attached to a farm, and those who do seasonal work find much the same demand year after year. On the other hand, dock-labour varies considerably.

Again, it has been shown above that many important industries produce little unemployment.

The percentages shown by these returns can, then, only be used to *measure* unemployment after a troublesome and hazardous estimate; their use is rather to form an *index* of unemployment, which shall reach its maxima and minima at the worst and best times respectively, and fluctuate much or little as the state of the labour market changes is unstable or steady. In the following paragraphs the percentages are used in this sense.

A study of the table on p. 156 shows that unemployment has fluctuated in periods which are nearly decennial, the worst years being 1858, 1868, 1879, 1886, 1893, and 1904, and 1908; in the last two decades the periods are less regular, a long spell of good employment (1886 to 1901) being followed by an abortive crisis in 1904, two fairly good years in 1906, 1907, and bad years in 1908–9.

On the whole, it cannot be said that unemployment as shown by these numbers has either increased or decreased over a long period; this would be seen better from a diagram than from the averages given, for these depend very much on what period is averaged.

The apparent severity of the worse periods arises from the preponderance of the engineering and shipbuilding trades, some branches of which fluctuate excessively. If these industries are subtracted, the remainder never shows a percentage so high as 6·1. The table on p. 153 above, for October 1909, shows also in that month for engineering and shipbuilding 13·8 % unemployed, and for other industries 4·0 %. Column D on p. 156 shows the effect of assuming that other industries as a whole are of the same numerical importance as these two.

4. Index of Unemployment. Labour Department. Percentage of Trade Unionists Unemployed.

	All industries for which Returns are available.				Shipbuilding and Engineering.		Other industries.		Decennial averages.
	A.	B.	C.	D.	A_1.	B_1.	A_2.	B_2.	E.
1851 . .	3·9	—	—	—	3·9	—	—	—	5·2 (9 years)
1852 . .	6·0	—	—	—	6·0	—	—	—	
1853 . .	1·7	—	—	—	1·7	—	—	—	
1854 . .	2·9	—	—	—	2·9	—	—	—	
1855 . .	5·4	—	—	—	5·4	—	—	—	
1856 . .	4·7	—	—	—	4·9	—	1·6	—	
1857 . .	6·0	—	—	—	6·1	—	2·3	—	
1858 . .	11·9	—	—	—	12·2	—	2·5	—	
1859 . .	3·8	—	—	—	3·9	—	1·4	—	
1860 . .	1·9	—	—	—	1·9	—	1·8	—	5·2
1861 . .	5·2	—	—	—	5·5	—	1·9	—	
1862 . .	8·4	—	—	—	9·0	—	3·1	—	
1863 . .	6·0	—	—	—	6·7	—	2·7	—	
1864 . .	2·7	—	—	—	3·0	—	·9	—	
1865 . .	2·1	—	—	—	2·4	—	1·2	—	
1866 . .	3·3	—	—	—	3·9	—	1·4	—	
1867 . .	7·4	—	—	—	9·1	—	3·5	—	
1868 . .	7·9	—	—	—	10·0	—	3·5	—	
1869 . .	6·7	—	—	—	8·9	—	3·0	—	
1870 . .	3·9	—	—	—	4·4	—	3·1	—	3·8
1871 . .	1·6	—	—	—	1·3	—	2·0	—	
1872 . .	·9	—	—	—	·9	—	1·0	—	
1873 . .	1·2	—	—	—	1·4	—	·9	—	
1874 . .	1·7	—	—	—	2·3	—	·9	—	
1875 . .	2·4	—	—	—	3·5	—	·9	—	
1876 . .	3·7	—	—	—	5·2	—	1·6	—	
1877 . .	4·7	—	—	—	6·3	—	2·5	—	
1878 . .	6·8	—	—	—	9·0	—	3·5	—	
1879 . .	11·4	—	—	—	15·3	—	6·1	—	
1880 . .	5·5	—	—	—	6·7	—	3·8	—	5·6
1881 . .	3·5	—	—	—	3·8	—	3·3	—	
1882 . .	2·3	—	—	—	2·3	—	2·4	—	
1883 . .	2·6	—	—	—	2·7	—	2·5	—	
1884 . .	8·1	—	—	—	10·8	—	3·5	—	
1885 . .	9·3	—	—	—	12·9	—	4·2	—	
1886 . .	10·2	—	—	—	13·5	—	5·6	—	
1887 . .	7·6	—	—	—	10·4	—	3·9	—	
1888 . .	4·6	4·9	—	4·1	5·5	6·0	3·4	2·3	
1889 . .	2·1	2·1	—	2·0	2·0	2·3	2·1	1·8	
1890 . .	2·1	2·1	—	2·1	2·4	2·2	1·6	2·0	4·4
1891 . .	3·2	3·5	—	3·4	4·4	4·1	1·8	2·7	
1892 . .	5·8	6·3	—	6·2	8·2	7·7	2·7	4·7	
1893 . .	—	7·5	—	7·7	—	11·4	—	4·0	
1894 . .	—	6·9	—	7·2	—	11·2	—	3·2	
1895 . .	—	5·8	—	6·0	—	8·2	—	3·8	
1896 . .	—	3·4	—	3·3	—	4·2	—	2·5	
1897 . .	—	3·5	—	3·4	—	4·8	—	2·1	
1898 . .	—	3·0	2·8	2·9	—	4·0	—	1·9	
1899 . .	—	2·4	2·0	2·0	—	2·4	—	1·7	
1900 . .	—	2·9	2·5	2·4	—	2·6	—	2·3	B. about 5·2
1901 . .	—	3·8	3·3	3·3	—	3·8	—	2·9	
1902 . .	—	4·4	4·0	4·2	—	5·5	—	2·9	
1903 . .	—	5·1	4·7	5·0	—	6·6	—	3·4	
1904 . .	—	6·5	6·0	6·4	—	8·4	—	4·4	
1905 . .	—	5·4	5·0	5·2	—	6·6	—	3·9	C. 4·8
1906 . .	—	4·1	3·6	3·7	—	4·1	—	3·3	
1907 . .	—	4·2	3·7	3·9	—	4·9	—	3·0	
1908 . .	—	8·1	7·8	8·6	—	12·5	—	4·8	
1909 . .	—	—	7·7	—	—	13·0	—	—	
1910 . .	—	—	4·7	—	—	6·8	—	—	
1911 . .	—	—	3·0	—	—	3·4	—	—	
1912 . .	—	—	3·2	—	—	3·6	—	—	
1913 . .	—	—	2·1	—	—	—	—	—	

Notes.

The numbers in the Columns A are partly based on the expenditures on unemployed benefits (Cd. 2337, p 91).

The numbers in Columns B are obtained as explained in the previous paragraph.

The numbers in Column C are the result of further information from certain Trade Unions, and are given in the *Labour Gazette*, January 1909, and January 1914.

The numbers in Column D are the simple averages of those in Columns B_1 and B_2, and are used to reduce the over-preponderance of engineering and shipbuilding in the unadjusted percentages (Cd. 4954, p. 223).

The averages in Column E are from Columns A and B or C.

It is important to notice that the more complete returns now obtained reduce the percentage unemployed by about ·4. In the comparison of statistics subsequent to 1908 with those of an earlier period, great care will be necessary. The effect of the newer figures is shown in Column C, from 1898 onwards. The alteration from this adjustment emphasizes the cautions already given as to the difficulty in the use of these percentages in *measuring* unemployment.

DETAILS AS TO UNEMPLOYMENT, A.S.E.

	1890.	1893.	Mean of the years 1887-1895.
Mean number of members in district, excluding superannuated .	6,344	6,934	6,507
Number of separate members unemployed for as much as 3 days some time during the year . .	1,356	1,832	1,929
Percentage of membership .	*21·4*	*26·4*	*29·7*
Average number unemployed at the *same* time during the year . .	134	706	403
Percentage of membership .	*2·1*	*10·2*	*6·1*
Aggregate number of working days lost through want of employment	40,825	215,874	123,166
Number per member . .	6·4	31·1	18·7
Average number of working days lost by those members who were unemployed for as much as 3 days in the year	30·1	117·8	63·1
Percentage number of members unemployed during the year for—			
Less than 3 days	78·6	73·6	70·4
3 days, and less than 4 weeks .	14·1	5·8	12·9
4 weeks, and less than 8 weeks	3·5	2·4	4·6
8 weeks, and less than 12 weeks	1·6	2·1	2·8
Over 12 weeks . . .	2·2	16·2	9·3

5. The annual averages hitherto given are obtained for each year by adding together the percentages shown at the end of each month and dividing by 12, a method which must give very nearly the same result as any more refined calculation possible. If the general average is 4 % it means that, so far as the group of occupations which are

ncluded is concerned, one person in 25 is on the average unemployed through the year.* This average may be made up by one man in each 25 having no work in the year, or each man losing one week in 25, or any distribution of unemployment between these extremes. We have very little information on this point, but the table on p. 157 (Cd. 2337, p. 101) shows the circumstances for the Amalgamated Society of Engineers. [See note on p. 160.]

In this case it is seen that about one quarter of the members bore the whole brunt of unemployment in the bad year 1893.

6. There are good and bad seasons in the year in nearly every occupation. These can be studied, if we average away the peculiarities of particular years as is done in the following table :—

PERCENTAGES OF UNEMPLOYMENT, SHOWING THE SEASONAL FLUCTUATIONS.

(Compiled principally from the *XIth Abstract of Labour Statistics.*)

		Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly mean.
All Trade Unions making returns.	1897	3·3	3·0	2·5	2·5	2·3	2·7	2·7	3·5	4·4	4·7	4·8	5·3	3·5
	1898	5·0	4·4	3·1	2·9	2·7	2·6	2·6	2·8	2·6	2·5	2·3	2·9	3·0
	1899	3·0	2·6	2·5	2·2	2·5	2·3	2·3	2·3	2·4	2·3	2·2	2·5	2·4
	1900	2·7	2·9	2·3	2·5	2·4	2·6	2·7	3·0	3·6	3·3	3·2	4·0	2·9
	1901	4·0	3·9	3·6	3·8	3·6	3·5	3·4	3·9	3·7	3·7	3·8	4·6	3·8
	1902	4·4	4·3	3·7	3·9	4·0	4·2	4·0	4·5	5·0	5·0	4·8	5·5	4·4
	1903	5·1	4·8	4·3	4·1	4·0	4·5	4·9	5·5	5·8	5·8	6·0	6·7	5·1
	1904	6·6	6·1	6·0	6·0	6·3	5·9	6·1	6·4	6·8	6·8	7·0	7·6	6·5
	1905	6·8	6·2	5·6	5·6	5·1	5·2	5·2	5·4	5·3	5·0	4·7	4·9	5·4
	1906	4·7	4·4	3·9	3·7	3·6	3·7	3·6	3·8	3·8	4·4	4·5	4·9	4·1
Averages for 10 years 1897–1906.														
All		4·6	4·3	3·75	3·7	3·65	3·8	3·75	4·1	4·3	4·35	4·3	4·9	4·1
Building (carpenters and plumbers) .		5·4	5·2	4·4	3·6	3·4	3·7	3·5	3·3	3·8	4·4	4·8	5·9	4·3
Engineering . .		4·5	4·4	4·0	3·8	3·6	3·7	3·7	4·0	4·2	4·4	4·5	5·3	4·2
Shipbuilding . .		8·0	6·8	6·1	6·3	6·0	6·2	6·1	6·8	8·1	9·2	9·9	9·8	7·4
Printing . . .		4·8	4·3	3·6	4·2	4·5	4·4	3·7	5·8	5·4	4·5	2·9	4·2	4·4
Furnishing . .		7·9	6·5	3·2	2·4	2·5	3·2	4·1	4·1	4·2	4·5	4·9	7·1	4·5

* Not exactly, for persons losing only a day or two are not registered, and, on the other hand, the names of those who get employment may not be taken off at once.

Here we see a distinct but small monthly fluctuation when all trades are merged together; the months from March to July are the best, December and January the worst. December or January is the worst month for the separate groups except printing, while the best month varies from March to August.

7. It is interesting and important to study these statistics in times of depression in the light of these seasonal fluctuations.

1907.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Engineering . .	3·2	2·8	2·9	2·8	2·9	3·4	3·5	3·8	4·1	4·5	4·7	6·4
Shipbuilding . .	8·8	7·5	7·1	6·6	6·7	6·4	7·2	9·3	11·7	11·6	12·8	14·2
Others . . .	3·9	3·7	3·3	3·0	3·1	3·4	3·3	3·4	3·8	3·8	4·2	4·8
All	4·2	3·9	3·6	3·3	3·4	3·6	3·7	4·0	4·6	4·7	5·0	6·1
1908.												
Engineering . .	5·8	5·9	7·1	8·6	9·5	10·5	11·1	12·0	12·3	12·7	13·0	14·0
Shipbuilding . .	15·1	20·0	21·5	23·2	26·1	20·6	22·2	25·2	26·6	26·3	25·2	24·7
Others . . .	5·1	4·8	4·9	5·2	5·4	5·6	5·3	5·7	5·4	5·5	5·4	5·5
All	6·2	6·4	6·9	7·5	7·9	8·2	8·2	8·9	9·4	9·5	9·1	*9·7**
1909.												
Engineering . .	13·1	12·8	12 6	12·4	12·5	12·1	12·1	11·3	10·8	10·3	9·5	9·8
Shipbuilding . .	23·0	22·5	22·2	23·3	23·4	23·6	23·9	23·3	22·4	21·5	19·3	16·9
Others . . .	5·4	5·0	4·9	4·7	4·3	4·5	4·4	4·5	4·4	4·0	4·0	4·2
All	*8·7*	*8·4*	*8·2*	*8·2*	*7·9*	*7·*[illegible]	*7·9*	*7·7*	*7·4*	*7·1*	*6·5*	*6·6*

* New basis of computation begins here.

These numbers were disturbed by strikes in the shipbuilding and engineering trades, which were settled respectively in May and September 1908; though persons actually on strike are not included, the influence is always felt beyond the nominal area. From and after December 1908, the figures for "all" are on the new basis named on pp. 156–7 above, and are about ·4 below the height they would otherwise have reached; a slighter effect is produced in the other lines.

It is very noticeable that the two industries of engineering and shipbuilding account for the apparent acuteness of unemployment in 1908 and 1909. The maximum for "others" is not much above the percentage 4·4 reached in 1904 (see p. 156).

In the previous table it is seen that the general percentage for all trades for November is equal to that for October and less than that for December; but in 1908 there was a fall in November and no change in December, hence October 1908 was relatively as well as absolutely the worst month for employment in general. During 1909 there was less than the usual fall in the early part of the year, but August and onwards showed an improvement instead of the usual seasonal increase of unemployment.

In engineering the changes from June 1908 to July 1909 are very nearly proportional to the ordinary seasonal change; no non-seasonal improvement is visible till August 1909. In shipbuilding similarly the first definite sign of improvement is in August 1909.

In addition to the unemployment which has been the subject of this chapter, work is, of course, lost from sickness and from strikes and lock-outs. The former varies little from year to year; the latter are dealt with on pp. 160–3.

8. In recent years the Ministry of Labour has published monthly in the *Labour Gazette* two sets of statistics which throw light on the amount of unemployment. The first shows the number of workpeople desiring employment on the "Live Register" of the Labour Exchanges. For short periods the changes in this number are significant, but over longer periods they are liable to be influenced by changes in the policy of the Ministry and in the attitude of workpeople to the Exchanges. The second shows, separately for a large number of industries and with totals for all insured, the number of males and females insured and the number of Unemployment Books lodged at the end of each month. The "lodging" of a book is necessary if its owner is to obtain unemployment payment. These figures are subject to modification with every change of the Insurance Acts, either in inclusion of new industries or in administration, but experience is showing that they afford a good measurement of unemployment. The statistics are explained by Mr. J. Hilton in the *Statistical Journal*, March 1923.

CHAPTER VIII

OTHER STATISTICS RELATING TO THE WORKING CLASSES

1. BESIDES the statistics of occupation, production, wages and employment already dealt with, there are several other statements relating to the working-class, most of which are summarized in the *Annual Abstract of Labour Statistics*. We will omit the statistics of profit-sharing, of industrial accidents, and of diseases of occupations, and deal briefly with the tables relating to trade disputes, Trade Unions and co-operation.

The statistics relating to strikes and lock-outs* are obtained directly from the employers and Trade Unions concerned during and at the end of the dispute. Apart from information as to the wages and normal hours of labour recognized before and after, and as to changes of any kinds made in the conditions of employment or working arrangements, the statistics collected relate to the causes and to the results of the disputes and to the methods by which they were terminated, to the number of persons directly or indirectly affected, and to the number of working-days lost.

By the number of persons *directly* affected is meant those who are actually on strike or locked-out; in the number *indirectly* affected are included "other workpeople employed at the establishments where the dispute occurred, and thrown out of work by the dispute." Clearly this latter category is arbitrary; if carpenters were the permanent servants of

* The information is given in much more detail in the Annual Reports on Strikes and Lock-outs.

a firm whose works were closed they would be classed as "indirectly" affected, whereas if they were hired through a contractor as required they would be equally affected by the loss of work, but would not be included. In fact the effect of a strike cannot be measured; members of all the industries, at home or abroad, who furnish material for the manufactures which are stopped or use their finished products, and at a later stage the great multitude of people who in general provide the strikers with commodities which they can no longer afford when their wages stop, are to a greater or less extent thrown out of employment; the effects of a strike spread through industry like ripples over a pool when a stone is dropped into it.

The number directly affected is rendered indefinite by the difficulty in distinguishing them on any definition from those indirectly effected. If weavers are on strike, the sizers and dressers may cease work either because they sympathize with the weavers' grievances, or because their work is useless when the looms are stopped, or because the employer locks out all hands. The effect is much the same, but in the first case they are "directly," in the others "indirectly" affected.

This difficulty ot definition cannot be got over, and therefore the statistics, and others based on them, can only be used as indication of the effect of disputes, and with due caution for comparing one year with another.

The number of days lost through a dispute is computed from the number of workpeople stopping work and the duration of the stoppage. This is a little fictitious, for there is no certainty that this number would have obtained work throughout if there had been no stoppage, and it is probable that either there will be extra work to do after the dispute, or that more work has been done in other places during the dispute, or that trade has been permanently displaced. These criticisms have yet more force when the loss of wages is computed, as is sometimes done unofficially in this country and officially in others.

In fact, the circumstances of strikes cannot be made the subject of exact statistics; we can only note in general terms whether they are becoming more or less acute as the years go on. The following table shows the principal statistics for the United Kingdom for 1893–1906.

	No. of disputes.	No. of work-people directly affected.	No. of work-people indirectly affected.	Aggregate duration in working-days, *i. e.* total number of days lost by persons directly or indirectly affected.*	Percentage number of disputes settled.		
					In favour of work-people.	In favour of employers.	Compromised or indefinite.
		000's	000's	0,000's			
1893	615	594	40	30,47	40	34	26
1894	929	257	68	9,53	35	36	29
1895	745	207	56	5,72	35	37	28
1896	926	148	50	3,75	41	33	26
1897	864	167	63	10,35	38	36	26
1898	711	201	53	15,29	33	32	35
1899	719	138	42	2,52	32	35	33
1900	648	135	53	3,15	31	34	35
1901	642	111	68	4,14	25	44	31
1902	442	117	140	3,48	24	47	29
1903	387	94	23	2,34	23	48	29
1904	355	56	31	1,48	17	51	32
1905	358	68	26	2,47	20	46	34
1906	486	158	60	3,03	31	37	32
1907	601	101	47	2,16	32	41	27
1908	399	224	72	10,83	20	44	36
1909	436	170	131	2,77	18	46	36
1910	531	385	130	9,89	25	37	38
1911	903	831	131	10,32	25	32	43
1912	857	1,233	230	40,91	27½	30½	42
1913	1,497	516	173	11,63	29	25	46

* This includes in each year days lost by persons through disputes which began in a previous year.

The high numbers of working days lost were mainly due in 1893 to the strike of coal-miners in the Federated Districts, in 1894 to the Scottish coal-miners' dispute, in 1897–8 to the engineers' dispute, in 1908 to the shipbuilders' and South Wales coal-miners' and cotton-operatives' disputes, in 1911 to the railway strike, and in 1912 to the coal strike.

The series in this table do not show any very definite trend,

nor any clear connection with the periods of good or bad trade or of rising or falling wages.

2. The statistics relating to Trade Unions have been for many years good and complete. In general, very careful accounts are kept in detail of membership, receipts and expenditure by the officials of the various unions, and are published periodically for the information of their members. These and special reports are easily available for the Registry of Friendly Societies and for the Labour Department, both of whom issue detailed reports. The more interesting details are summarized for 100 principal Trade Unions in the Abstract of Labour Statistics. Full details are given in the Report on Trade Unions (*e. g.* Cd. 6109).

	Number and membership of all Unions from which information is received.		100 Principal Unions.										
						Expenditure on various "benefits."							
	Number.	Membership.	Membership.	Funds at end of year.	Income.	Unemployed.	Dispute.	Sick & Accident.	Superannuation.	Funeral.	Miscellaneous.	Working and other expenses.	Total expenditure.
		000's	000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
1899	1310	1,861	1,164	3,226	1,835	187	120	287	174	90	69	327	1,253
1900	1302	1,972	1,210	3,733	1,950	263	140	308	184	95	92	362	1,443
1901	1297	1,979	1,219	4,139	2,050	328	210	326	193	95	101	387	1,644
1902	1267	1,966	1,217	4,426	2,094	433	221	341	217	95	95	405	1,807
1903	1255	1,942	1,206	4,612	2,109	517	176	364	238	93	96	439	1 923
1904	1229	1,911	1,203	4,680	2,124	660	118	388	265	96	102	426	2,056
1905	1228	1,934	1,220	4.830	2,228	529	214	403	286	93	116	432	2,078
1906	1250	2,129	1,307	5,222	?,364	429	154	415	306	99	105	465	1,972
1907	1243	2,425	1,471	5,668	2,518	469	138	434	323	105	113	486	2,072
1908	1218	2,389	1,451	5,201	2,767	1,026	606	467	355	107	138	534	3,234
1909	1199	2,369	1,437	5,079	2,585	952	156	440	376	107	145	529	2,707
1910	1195	2,446	1,472	5,153	2,716	702	352	419	403	104	138	524	2,642
1911	1204	3,019	1,821	5,595	2,952	457	318	436	412	113	197	578	2,510
1912	1149	3,288	2,000	5,002	3,230	593	1,375	440	425	119	163	703	3,823
1913	1135	3,987	Particulars not yet available.										

The relatively small amounts spent on Dispute Benefit as contrasted with the amounts on unemployment, superannuation, sickness and accidents are very noticeable.

At the end of 1911 the funds in possession of these unions was £5,595,000, only about $2\frac{1}{4}$ times the annual expenditure. Of this total £1,699,000 was held by 16 unions connected

with mining and quarrying, and £1,420,000 by 15 unions in the metal, engineering, and shipbuilding trades.

3. The Friendly Societies have in the aggregate very much larger funds and a much greater membership than Trade Unions. The methods, objects and importance of the 29,000 societies registered vary so much that the gross totals show very little, but are given to show the numerical importance of these institutions. The 14 societies which are grouped together in the following table have many branches and are specially important.* The growth has been uninterrupted, so that it is not necessary to give statistics for intermediate years.

All Friendly Societies.	1897.	1907.	1912.
No. of registered societies	29,381	29,310	29,317
Membership	10,934,000	15,983,264	15,681,013
Accumulated funds.	£35,736,000	£57,128,168	£67,110,688

Aggregate of 14 important Societies.	1897.	1907.	1912.
Adult male membership	2,255,000	2,678,000	3,197,000 †
Yearly income	£3,969,000	£5,129,000	£5,585,000
Total accumulated funds.	£18,447,000	£28,498,000	£34,369,000
Total benefits of all kinds	£2,479,000	£3,436,000	£4,302,000
Sick benefit	£1,809,000	£2,499,000	£2,948,000
Funeral benefit	£383,000	£466,000	£466,000

The 14 societies own thus half of the total accumulated funds shown for all the societies. The great part of the expenditure is on Sick Benefit. It is not easy to deal with the statistics of Friendly Societies in further detail in a compact form, and the figures just given are rough. Considerable detail is to be found in the *Annual Reports of the Chief Registrar of Friendly Societies.*

* The five largest, viz. the Manchester Unity of Oddfellows, the Ancient Order of Foresters, I.O. Rechabites (Salford Unity), the Hearts of Oak, and the National Deposit Friendly Society have between them 2,511,000 members.

† "State only" members are omitted in Hearts of Oak (1913).

4. The Registry of Friendly Societies also receives information as to Building Societies and as to Co-operative Societies, which (together with other information specially collected) is summarized in the *Annual Abstract of Labour Statistics.* The former are not confined to the working-class, and the statistics are not easy to interpret. The latter hold a very important part of the aggregate of working-class savings, and no small proportion of working-class expenditure is accounted for in their statistics of sales. The following table contains some summary statistics of these societies:—

All Co-operative Societies in the United Kingdom for which information is received.	1898.	1903.	1908.	1912.
	000's	000's	000's	000's
Number of Members	1,596	2,089	2,526	2,898
Capital, Share	£19,280	£26,601	£33,082	£38,403
Capital, Loan	£4,983	£7,994	£10,772	£13,742
Amount of Sales	£70,347	£99,122	£128,752	£151,015
Sales by Retail distribution societies	£42,582	£57,513	£69,786	£83,607
Sales by the English Wholesale society's distributive departments	£12,575	£19,333	£24,903	£31,372
Sales by the Scottish Wholesale society's distributive departments	£4,692	£6,395	£7,531	£8,964

For comparison with these figures it may be added that the total paid in wages in the United Kingdom is estimated roughly at about £800,000,000 per annum. Of course sales are not exclusively to the working-class.

The statistics of the last three paragraphs suggest a very interesting investigation, beyond the scope of the present work, as to the aggregate savings of the working-class. Sir E. W. Brabrook (then Chief Registrar of Friendly Societies) devoted his Presidential Address to Section F of the British Association in 1903 to this subject (*Report*, 1903, pp. 729–740), and gave more recent information in a paper "On the Progress of Friendly Societies, etc.," *Statistical Journal*, 1915, p. 414, especially p. 432.

5. A great deal of attention has been given from time to time in various countries to working-class "budgets," which show the cost and amount of the various commodities on which wages are spent. The information is always collected first-hand from the workman or his wife, and it is not easy to secure accurate accounts either of income or expenditure; for to include clothes and occasional earnings these accounts should be spread over a long period, an undertaking that requires intelligence, time and attention to minutiæ on the part of the informant. Often, in fact, the budgets do not exactly balance; expenditure on drink and luxuries tends to be underestimated, and in the end the returns apply only to specially thrifty households. So far as the items contained in the following table are concerned these objections do not apply. The tables are taken from the Reports on the Cost of Living of the Working Class in the United Kingdom, France and Germany (Cd. 3864, 4032 and 4512); these volumes contain a great wealth of detailed information as to wages, prices, rents and conditions in a large number of towns in the countries named. The relative levels of wages, rents and prices are compared by means of a system of index-numbers which presents many statistical difficulties, and the results based on them should be criticized closely in relation to the data. As soon as we attempt to compare the well-being of two groups of people we find that statistics of incomes, prices and methods of expenditure only take us part of the way; habits, desires, thriftiness and skill in domestic economy vary greatly from class to class and from nation to nation, and cannot be reduced to statistical measurement; but the data in these volumes make possible vivid descriptions of the economic life of typical working-class families throughout the towns of the three nations.

Similar Reports (Cd. 5065 and 5609) have been published as to Belgium and U.S.A.

The figures given in the following tables, however, present few statistical difficulties.

AVERAGE WEEKLY COST AND QUANTITY OF FOOD CONSUMED BY URBAN WORKMEN'S FAMILIES, UNITED KINGDOM, 1904.

Limits of weekly income	Under 25s.	25s. and under 30s.	30s. and under 35s.	35s. and under 40s.	40s. and above.
Average weekly family income	21s. 4½d.	26s. 11¾d.	31s. 11¼d.	36s. 6¼d.	52s. 0½d.
Average number of children living at home	3·1	3·3	3·2	3·4	4·4
			COST.		
	s. d.	s. d.	s. d.	s. d.	s. d.
Bread and flour	3 0½	3 8¾	3 3½	3 4¼	4 3¾
Meat (bought by weight)	2 8	3 4¾	4 3½	4 5½	5 10½
Other meat and fish	0 7½	0 8¾	0 10	1 0	1 4
Bacon	0 6¾	0 9	0 10¼	0 11½	1 3¾
Eggs	0 5¾	0 8½	0 11	1 0	1 4¾
Fresh milk	0 8	0 11¾	1 3¼	1 4¼	1 7¾
Cheese	0 4¾	0 5½	0 6	0 6	0 8
Butter	1 2	1 7	1 10¼	2 0	3 0½
Potatoes	0 8¾	0 9¾	0 10½	0 10¼	1 1¾
Other vegetables and fruit	0 4¾	0 7	0 10	0 11¾	1 3¾
Rice, tapioca and oatmeal	0 4½	0 5	0 6	0 5¾	0 7
Sugar	0 8	0 10	0 10¾	0 11¼	1 3
Tea	0 9¼	0 11½	1 0¾	1 1½	1 5
Coffee and cocoa	0 2	0 3¼	0 3½	0 4¼	0 5½
Jam, etc.	0 4¼	0 5¼	0 6	0 6½	0 8¾
Other items	1 4	1 7¾	2 0	2 5	3 2¼
Total expenditure on food	14 4¾	17 10¼	20 9¼	22 3½	29 8
Expenditure on bread and flour, as % of food cost	21	19	16	15	15
Expenditure on fish, meat and bacon, as % of food cost	27	27	29	29	28
Expenditure on all food, as % of income	67	66	65	61	57
			QUANTITIES.		
	lbs.	lbs.	lbs.	lbs.	lbs.
Bread and flour	28·4	30·0	29·4	30·0	37·8
Meat (bought by weight)	4·4	5·3	6·3	6·4	8·2
Bacon	·9	1·1	1·2	1·4	1·8
Cheese	·7	·7	·8	·8	1·0
Butter	1·1	1·5	1·7	1·9	2·8
Potatoes	14·0	15·8	16·1	15·9	19 9
Tea	·48	·55	·57	·59	·72
Sugar	3·9	4·6	4·8	5·2	6·7
	pints.	pints.	pints.	pints.	pints.
Fresh milk	5·5	7·7	9·8	10·3	12·6

French Towns. 1907–8

Limits of weekly income	Under 20*s.*	20*s.* and under 25*s.*	25*s.* and under 30*s.*	30*s.* and under 35*s.*	35*s.* and under 40*s.*	40*s.* and above.
Average weekly family income	17*s.* 9½*d.*	22*s.* 11*d.*	27*s.* 7¾*d.*	32*s.* 4½*d.*	37*s.* 8¾*d.*	52*s.* 11*d.*
Average number of children living at home .	1·6	1·8	1·8	1·9	2·1	2·9
	Cost.					
	s. *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*
Bread	2 9	2 11¾	3 1½	3 4¼	3 8	4 8½
Potatoes	0 7¾	0 7½	0 7¾	0 8	0 9	0 11¾
Meat, bacon, etc. . .	3 2	4 2½	5 0¾	6 2¼	6 11	9 4¼
Sugar	0 4¼	0 5¼	0 5¼	0 6	0 6¼	0 7¾
Other items	4 2¾	5 8	6 11¼	8 0¼	9 1	12 3¼
Total expenditure on food	11 1¾	13 11	16 2½	18 8¾	20 11¼	27 11½
Expenditure on bread, as % of food cost . .	24	21	19	18	18	17
Expenditure on meat, etc., as % of food cost .	28	30	31	33	33	33
Expenditure on all food, as % of income . .	63	61	59	58	56	53
	Quantities.					
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Bread	22·9	24·1	24·6	26·2	27·6	35·9
Potatoes	14·7	12·3	13·9	14·6	15·8	20·5
Meat	4·2	5·5	6·5	7·8	8·6	11·5
Sugar	1·2	1·5	1·5	1·7	1·8	2·2

German Towns. 1906–7

Limits of weekly income	Under 20*s.*	20*s.* and under 25*s.*	25*s.* and under 30*s.*	30*s.* and under 35*s.*	35*s.* and under 40*s.*	40*s.* and above.
Average weekly family income	17*s.* 7¾*d.*	22*s.* 8½*d.*	27*s.* 1*d.*	31*s.* 10½*d.*	36*s.* 8*d.*	48*s.* 8½*d.*
Average number of children living at home .	2·4	2·3	2·5	2·5	2·8	3·8
	Cost.					
	s. *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*
Bread and flour . .	2 5½	2 7¾	2 10½	3 0½	3 5	4 6¼
Potatoes	0 9¾	0 10	0 10¼	0 10½	0 11¾	1 2½
Meat, bacon, etc. . .	3 11	4 4½	5 0½	6 0	6 8¾	8 11¾
Sugar	0 4¼	0 4½	0 4¾	0 5	0 5¼	0 6½
Other items	4 7	6 4¾	7 8¼	8 6¼	9 7	12 1½
Total expenditure on food	12 1½	14 7½	16 10¼	18 10¼	21 1¾	27 4½
Expenditure on bread and flour, as % of food cost	20	18	17	16	16	16
Expenditure on meat, etc., as % of food cost.	32	30	30	32	32	33
Expenditure on all food, as % of income . .	69	64	62	59	58	56
	Quantities.					
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Bread and flour . .	22·0	23·5	25·0	26·1	29·8	38·2
Potatoes	28·1	25·6	24·0	23·8	24·6	33·5
Meat, bacon, etc. . .	4·9	5·6	6·2	7·4	8·2	10·8
Sugar	1·8	1·8	2·0	2·0	2·1	2·7

It must be remembered that these tables do not show the relative levels of wages in the three countries, but only the distribution of expenditure of wages of given amounts. The general agreement between the three tables is remarkable; in each country the proportion spent on food falls by about one-sixth part as incomes increase from the lowest to the highest; the *proportion* of the food expenditure devoted to meat is nearly the same for all incomes, while the *actual* expenditure increases. A difference is noticeable as to the consumption of bread and flour; in the United Kingdom it is from 28·5 to 30 lbs. except for the large families shown in the last column; in France and in Germany, on the other hand, the lower the income the less bread consumed per head. Among many interesting details we may notice the large amount of sugar consumed in the United Kingdom.

6. We have not dealt with Statistics of *Pauperism*, because they are likely to be very misleading, from incomplete and faulty definition, unless handled with special care and knowledge.

Education statistics are plentiful and accessible in the reports of the Board of Education.

Statistics relating to *Old Age Pensions* must be sought for in the Report of the Commissioners of Customs and Excise (*e. g.* Cd. 6993).

Copious information as to *Unemployment Insurance* is given monthly in *The Labour Gazette*, and a Report dealing with the whole subject will probably be issued annually. The second Annual Report of *National Health Insurance* was issued in 1914 (Cd. 7496).

CHAPTER IX

INCOME AND CAPITAL

1. By *Total National Income* is generally meant the aggregate of the incomes (including earnings) of the persons composing a nation; income is taken as meaning the money, or money value of goods, coming into a person's possession during a year for his own use (subject to rates and taxes), after all expenses connected with obtaining it are subtracted. The earnings of the working-classes, discussed in Chapter VI are thus measured, and incomes are assessed for income-tax on the basis of this definition.

It is doubtful whether a perfectly definite meaning can be attached to Total National Income. The sum of money nominally representing it of course does not actually exist; a great part of income is actually received in the form of cheques which are exchanged for services, and the total is more correctly the total estimated value of services rendered to, or commodities consumed by, the members of the nation, together with the addition to savings, that is to capital goods. In such a total are included the services of an agricultural labourer at £3 per month and of a physician at the same price for a short visit, the value of a day's sojourn at a hotel and the equal value of 60 quartern loaves of bread or 80 oz. of tobacco. The utility of £1 to a person is in general the less the greater his income, and the total utility of all incomes depends on how they are distributed among persons. On the other side, the value of services and commodities depends on the demand for them. In fact, the hundreds of millions of pounds which make the aggregate are not a homogeneous total and cannot be used for processes of averaging without analysis. To say that the average income of the inhabitants

of the United Kingdom is £40 is nearly meaningless, except as an arithmetical entity for use in arithmetical processes. The total depends on the existing method, and the momentarily resulting scale, of valuing various services and commodities; the scale is continually changing, and the total would easily be affected, for example, by a redistribution of income by taxation or under a socialistic *régime*.

Nevertheless, the total and resulting averages can be used for comparing total or average income or wages through a period so short that during it no great changes in valuation or in distribution have taken place.

2. The aggregate of the earnings of the wage-earning class is generally estimated by calculating the average annual earnings of men, women and children from the statistics described in Chapter VI, and multiplying these averages by the numbers of persons occupied, as indicated by the census. There is much that is hazardous in this method, but it seems probable that the aggregate of net earnings received by persons working for hire (including a valuation for payments in kind, etc.), was *circa* 1913 about £800,000,000 annually in the United Kingdom. This, of course, ignores completely the value of the unpaid domestic work done by women for themselves or their families or relations, and of many other unpaid services.

The aggregate of the incomes of those whose income is over £160 per annum is estimated from the income-tax returns at rather over £900,000,000 in 1913, as will be presently shown. This total includes earned and unearned income. A very small fraction of this has already been included under the former total of earnings, for some workmen pay income-tax.

Besides these two sums there are the incomes of those who neither work for wages nor receive as much as £160 annually as incomes. The total was estimated to be between £300,000,000 and £370,000,000 in 1909 by a Committee of the British Association (see *Statistical Journal*, Dec. 1910, for the report).

The aggregate of incomes of all kinds is thus estimated at

about £2,100,000,000 in the year 1913, but this total must be regarded as subject to considerable error, perhaps as much as 10 %.

Estimates on a similar basis show—

	Aggregate Income.	Population of United Kingdom.
	£000,000's.	00,000's
1860	750	28,8
1870	1,000	31,3
1880	1,200	34,6
1890	1,450	37,5
1900	1,750	41,2
1908	1,900	44,1
1913	2,100	46,0

These numbers are rough and uncertain, but they are better than no estimates, and can be used for such purposes as comparing the burden of taxation at different periods.

It must be remembered that the purchasing power of money diminished between 1860 and 1874 (see Chapter IV), increased till 1895, and fell again till 1907.

3. The statistical tables in the Annual Reports of the Inland Revenue Commissioners are full of pitfalls even for the wary. Their general nature can be well shown by the report for the Fiscal Year ending April 1, 1912.*

1912.	Gross income brought under the review of the department.	Exemptions in respect of small incomes.	Abatements.	Children.	Life insurance premiums.	Charities, Hospitals, etc.	Repairs—Lands and houses.	Wear and tear of machinery.	Other allowances and income on which tax was irrecoverable.	Total deductions.	Net income on which tax was received.
Schedule	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's	£ 00,000's
A	277,3	35,9	11,6	3	1,1	8,4	42,4	—	17,4	117,1	160,2
B	17,5	11,5	1,5	—	—	—	—	—	7	13,7	3,8
C	49,5	1,2	2,2	—	1	9	—	—	1,2	5,6	43,9
D	598,6	10,4	66,1	2,9	6,4	4,3	—	26,4	36,6	153,1	445,5
E	127,2	5	47,0	2,2	4,2	—	—	—	6,1	60,0	67,2
Totals	1070,1	59,5	128,4	5,4	11,8	13,6	42,4	26,4	62,0	349,5	720,6

* Cd. 4226.

Schedule A includes profits from the ownership of lands (£52,000,000), and houses with sites (£234,000,000). For the former one-eighth, for the latter one-sixth, of the profits have been excused from taxation since 1894 for repairs.

Schedule B consists of profits from the occupation of land; these are simply assumed to be for purposes of taxation one-third of the annual value of the farm, etc. The actual profits are not stated at all except by a very small number of farmers who elected to be assessed under Schedule D. The great part is exempted or abated.

Schedule C consists of profits from British (£14,500,000), Indian (£9,800,000), Colonial (£15,300,000), and Foreign *Government* (£9,900,000) securities.

Schedule D contains profits from "Businesses, Concerns, Professions, Employments (except those contained in Schedule E), and certain interest." Railways in the United Kingdom (£46,000,000), Mines (£18,000,000), Gasworks, Ironworks, Waterworks and Canals (£24,000,000), Indian, Colonial and Foreign Securities, Coupons and Railways (£69,000,000), and some other small items, are separately distinguished; but the great bulk of the Income in this Schedule (£422,000,000, out of £599,000,000 gross) is lumped together. The deduction allowed for wear and tear of machinery has increased considerably in recent years.

Schedule E is entirely composed of the salaries of Government officials and officials of corporations and public companies.

4. Incomes, which are under £160 from all sources taken together, are exempt from taxation. The total (£59,500,000) in the column in the above table shows only the sum that accidentally came to the notice of the Commissioners, some of which was charged with income-tax collected at the source; on this in 1911–12 £1,087,000 was repaid, while on the remainder no tax at all was paid. This total merely shows that as much as £59,500,000 is received annually by persons whose incomes are below £160, and of this £37,000,000 under Schedules A and C is from property; but it evidently does

not show the aggregate of such incomes, for no tax is paid on and no notification is made to the commissioners as to the majority of small salaries.

Abatements are made on all incomes proved not to exceed £700; that is, a certain sum, varying from £160 for incomes not exceeding £400 to £70 for incomes exceeding £600 and not exceeding £700, is deducted by the surveyor of taxes from the total income, and tax is charged only on the remainder. To obtain an abatement it is necessary to state the amount of total income, as defined by the Commissioners, from all sources. For the year 1911–12 the following claims were allowed:—

Range of incomes.	Amount of abatement on each income.	No. of abatements.	Total amount of abatements.
£	£		£000's.
160–400	160	689,352	110,296
400–500	150	72,554	10,883
500–600	120	44,686	5,362
600–700	70	27,257	1,908
		833,849	128,449

The numbers of abatements allowed during 15 years have been:—

Year.	Total No. of abatements.	Full rate of tax.	
	000's	s.	d.
1898–1899	543	0	8
1899–1900	577	0	8
1900–1901	601	1	0
1901–1902	636	1	2
1902–1903	664	1	3
1903–1904	696	0	11
1904–1905	709	1	0
1905–1906	725	1	0
1906–1907	737	1	0
1907–1908	764	1	0
1908–1909	780	1	0
1909–1910	782	1	2
1910–1911	808	1	2
1911–1912	834	1	2
1912–1913	864	1	2

The rapidity of this increase is supposed to show that, as the income-tax has been increased, fewer persons have neglected to take the necessary trouble to substantiate their claim; and it is supposed that there are still many persons with incomes between £160 and £700 who do not appear in this table. The total number of persons with such incomes was estimated * to be about 840,000 in 1904–5, of whom only 709,000 claimed abatement.

Allowance is made for Life Insurance Premiums, not exceeding one-sixth of the net personal income, paid by the tax-payer on his life or on his wife's life.

A parent whose income is less than £500 is entitled to relief of tax on £10 for each child under 16 years old.

Charities, Hospitals, Friendly Societies and some other institutions are relieved of income-tax.

5. The statistics of Gross Income are often quoted as showing the growth of income as a whole, but they include much that is not income, and the allowances have increased considerably in recent years. The best plan of statement seems to be as follows :—

1911–12.	£00,000's	
Income on which tax was received and not refunded .	720,6	£866,000,000 personal † income above £160.
Abatements on incomes from £160 to £700 and on children	128,4 + 5,4	
Insurance payments . .	11,8	
Charities, hospitals, etc. .	13,6	Not personal income.
Repairs	42,4	Not income.
Wear and tear	26,4	
Other allowances . . .	62,0	Mostly not income.

† This includes, however, incomes of clubs and similar bodies and a small amount of income of municipalities and other local authorities.

Working through the income-tax reports, and allowing for the deductions on the present basis as accurately as possible in earlier years, we have the following table :—

* Report of Committee on Income Tax (H. of C., 365 of 1906), p. 229.

ESTIMATED AGGREGATE PERSONAL INCOME ABOVE EXEMPTION LIMIT.

[Exemption limit, 1860–1876, £100; 1877–1893, £150; from 1894, £160.]

Fiscal year.	£000,000's	Fiscal year.	£000,000's
1859–1860	254	1885–1886	498
1860–1861	254	1886–1887	496
1861–1862	244	1887–1888	504
1862–1863	273	1888–1889	518
1863–1864	285	1889–1890	544
1864–1865	309	1890–1891	568
1865–1866	326	1891–1892	569
1866–1867	335	1892–1893	572
1867–1868	341	1893–1894	562
1868–1869	344	1894–1895	553
1869–1870	355	1895–1896	567
1870–1871	385	1896–1897	587
1871–1872	399	1897–1898	611
1872–1873	430	1898–1899	641
1873–1874	461	1899–1900	663
1874–1875	482	1900–1901	695
1875–1876	490	1901–1902	715
1876–1877	473	1902–1903	720
1877–1878	476	1903–1904	732
1878–1879	470	1904–1905	738
1879–1880	462	1905–1906	753
1880–1881	468	1906–1907	764
1881–1882	481	1907–1908	799
1882–1883	493	1908–1909	824
1883–1884	507	1909–1910	822
1884–1885	505	1910–1911	838
		1911–1912	866

Average income has risen about as fast as the exemption limit during the whole period. The number of income-tax payers is not, and cannot be, known directly from the report; it was estimated at about 1,000,000 in 1906.* The table just given probably shows the general features of the growth of that part of the national income which is subject to income-tax with fair accuracy, and the rate of growth may accurately be deduced over quite short periods, if no exceptional event occurred in them; but there are many difficulties, some still

* Answer in the House of Commons, May 1914. Number of Payers 950,000 in 1903–4; " " 1,190,000 in 1913–14 } *Westminster Gazette*, May 14, 1914.

the subject of controversy, in such an estimate, which we will enumerate without discussion :—

The amount shown for a year (say 1906–7) is the total income in respect of which the tax was paid or remitted in the year ending April 5 (1907). Under Schedule D more than half (£373,000,000 profits on businesses not otherwise detailed) is assessed on the average profits of the preceding three years (presumably 1903, 1904, 1905), mines (£16,000,000) are assessed on the average of the preceding five years, and about £54,000,000 more on the profits on the preceding year. The whole assessment for 1906–7 may be regarded as relating to a short period whose centre is the Calendar year 1905; the whole table should be set back, therefore, about a year, and the peculiarities of individual years are averaged away. Thus the high profits in 1907 continued to have effect on the figures till the year 1912–13, for which the Report was published in the autumn of 1914!

It is generally supposed that greater vigilance and new powers of the surveyors of taxes have disclosed from 1907 onwards considerable amounts of income which had hitherto evaded taxation. If this is so, the amounts for years prior to 1907 should be somewhat raised for comparison with 1907 and later years.

It is believed that some part of the income which is received from abroad, and is liable to taxation, successfully evades taxation; naturally this amount can only be guessed. It is not improbable that in recent years the net of the commissioners has become finer and wider, and that less and less escapes. Actually £80,000,000, paying tax was identified in 1906–7 as income from abroad, and besides this there are other large sums included in Sch. D (52nd Report, pp. 163–5). If less escape than in former times, earlier figures should again be increased for comparison with more recent.

To get the total income above £160 it would be necessary to add an estimate for such income from abroad as escapes, and also an estimate for profits of trades and professions which are generally believed to be on the whole under-

valued. £80,000,000 is a current guess for these two amounts together, but in fact there are practically no data for an estimate.

6. Sometimes the gross returns for income-tax have been placed alongside the returns of Changes of Wages, discussed in Chapter VI above, and the conclusion drawn that income grows continuously while wages have been nearly stationary from 1900–8. Wages did in fact lose relatively to incomes in this period, but the relative rates of growth cannot be shown from the statistics, for the following reasons:—

The wage-changes published only apply to a small part of the working population, and afford no test of the general growth of wages (pp. 141 *seq.*, above).

The most recent statistics available were for the income assessed * for 1912–13 (and even these are incomplete), and these belong to 1911 rather than to any other year.

The income-tax returns cannot be allotted to any one year.

The relation of gross to net income has changed.

The collection of the tax has recently been more thorough.

The total net income, as shown in the table above (p. 177), *naturally grows* 1 % *per annum* with population, while the wage-changes have no relation to population.

The year 1900, which is frequently taken for comparison, was a year of exceptional inflation for wages, but is a normal year in the income-tax returns.

The following table shows the present writer's estimate of the change of average wages, and of the average income of the income-tax payer, each expressed as a percentage of the level in 1880. The former is from p. 147 above, the latter is based principally on the method of the table † on p. 177, with

* It is true that the net *receipt* in the year 1913–14 is known, and is 6½% higher than that for 1912–13; but the net receipt in a year and the net produce for the year are not the same, for considerable sums are paid more than a year late.

† See *Economic Journal*, 1904, p. 459; and for another view of the same problem, see *The Change in the Distribution of Income*, Clarendon Press, 1920, by the present author.

the years adjusted and allowance made for the growth of population, and with some other modifications based on the discussion in paragraph 5 above. The method is open to a great many fairly obvious criticisms.

INDEX-NUMBERS OF INCOMES AND WAGES.†

	Wages.	Incomes.		Wages.	Incomes.		Wages.	Incomes.
1880	100	100	1891	115	103	1902	126	118
1881	100	100	1892	115	100	1903	125	118
1882	103	103	1893	115	100	1904	123	119
1883	103	101	1894	115	101	1905	123	120
1884	103	100	1895	115	103	1906	126	124
1885	101	97	1896	115	107	1907	133	127
1886	100	97	1897	116	109	1908	131	125
1887	101	99	1898	120	111	1909	129	127
1888	104	103	1899	123	113	1910	129	131
1889	110	107	1900	130	117	1911	131	133
1890	114	105	1901	128	117	1912	135	138

† There is no allowance for the cycle of unemployment in this table. Such allowance would raise the numbers in some years and lower them in others, without affecting the general run. The averaging of the incomes under Schedule D also merges together good and bad years for the income index-number.

In making this computation care has been taken to exclude the same *proportion* of income as exempt (see *Economic Journal*, 1904, p. 460), so that the intermediate class who are not wage-earners but have small incomes are excluded from the calculation on the same proportionate basis throughout.

7. Material is accumulating for the estimation of income earned as opposed to unearned (or derived from capital). When a person or firm uses his own capital his profits are counted as earned. From the year 1907–8 any individual whose total income did not exceed £2,000 is entitled to such relief as would reduce the rate on his earned income to 9*d.* in the £. From 1909–10, when the general rate of income-tax was raised from 1*s.* to 1*s.* 2*d.*, a concession was also made to persons whose incomes were between £2,000 and £3,000, so as to bring the rate on the part of such income as was earned down to 1*s.*

The number of *persons* who have paid tax at the lower rate has not been published, except for the statement that about 750,000 persons paid at the rate of 9*d*. in 1907 and 1908.

The amount of income taxable at the various rates, calculated before abatements for incomes under £700, for Insurance, or for children were made, was :—

AMOUNT OF INCOME £000,000's.

	Taxable at		Taxable at	Total.
	9*d*.		1*s*.	
1907–8	188		612	800
1908–9	197		627	824
	9*d*.	1*s*.	1*s*. 2*d*.	
1911–12	221	13	632	866
1912–13	232	14	661	907

It is not easy to connect these figures with the table on p. 173. The taxable incomes under Schedules A and C, which are unearned, amount to £216,000,000. To get the amount taxed at 1*s*. 2*d*. in 1911–12 we must find £416,000,000 from B, D, and E. B (farmers) gives practically nothing, and only a small part of E (official employees) belongs to persons with more than £3,000 a year. Under E we find that Public Companies, Local Authorities, and tax deducted by bankers, etc., together account for £375,000,000 gross, from which a large deduction must be made for wear and tear and other allowances. We have still to find a large sum of unearned income in the hands of those who also earn, and either make over £3,000 a year in all or do not claim to be taxed at the reduced rate. The whole income from capital is less than the total taxed at 1*s*. 2*d*. (or whatever is the unreduced rate) by whatever earned income is thus included, but there are on the other side considerable sums of unearned income exempt from tax because the owner has less than £160 a year, or paying at the lower rate because the owner is employing the capital himself. Thus it is by

no means an easy matter to separate aggregate income into definite categories, one containing all due to ownership, the other all due to work.*

8. The statistics relating to the super-tax are distinguished in the reports of the Inland Revenue Commissioners from those for the income-tax. In them we have details of the income of those persons whose incomes are over the super-tax limit. These, together with the statistics of abatements, show that of the 1,100,000 income-tax payers of 1912–13, at least 864,000 had less than £700 and at least 12,200 over £5,000. It is a difficult problem to make a more detailed estimate of the distribution of income by amount.

9. The aggregate capital owned by the individuals of the nation can be estimated either by capitalizing the "unearned" income, or from the records of estates paying death duties. The first method was used by Sir R. Giffen in his essay on "Recent accumulations of capital in the United Kingdom,"† 1878. The latter has been the subject of much recent work. Unfortunately, it is extremely difficult to reconcile the results reached by the two methods.

To use the records of estates, assessed for Estate Duty, it is necessary to estimate the number of estates in existence in relation to the number which pass per annum. The best estimate appears to be that by Mr. B. Mallet and Mr. H. C. Strutt,‡ who, by tabulating the values of the estates according to the age of the deceased, and multiplying by the reciprocal of the death-rate age by age, arrive at the multiplier 28; that is, they conclude that 28 times the value of estates passing in one year gives the total value of such estates in existence. A higher multiplier had been used in previous estimates, but this neglected the important fact that estates as a whole increase with the age of their possessors.

* On the whole subject see Report of Committee on Income Tax, H. of C., 365 of 1906, and Messrs. Mallet and Strutt's paper alluded to below, and *British Incomes and Property*, J. C. Stamp.

† *Essays in Finance*. Also in *Statistical Journal*, 1878.

‡ *Statistical Journal*, July 1915, "The Multiplier and Capital Wealth."

The following table shows the results of this estimate in relation to the income-tax returns. It is modified from that on p. 220 of the Report of the Committee on the Income Tax.

UNITED KINGDOM.

Assessed values of estates reviewed for estate duty. Average of 10 years, 1894–1904, multiplied by 1·1 to bring up to 1904–5.		Presumed assessed value of all estates, 28 times previous column.	Corresponding income 1904–5, from income-tax returns. [Allowances deducted from gross income, but not insurance, abatements or exemptions.]		Averages deduced from previous columns. Rate of Interest per cent.	Number of years' purchase.
	Millions	Millions		Millions		
Stocks, companies, mortgages, bonds, mines and quarries	£127	£3,556	Companies, etc.	£265	7·5	13½
Agricultural land, timber, building land	26	728	Lands	41	5·6	18
Houses, and all rents that can possibly be connected therewith	63	1,764	Buildings	153	8·7	11½
	£216	6,048		£459	7·6	13
Goodwill, share in firms, book debts, stock-in-trade, half cash at bank	29	812	Unknown.			
	£245	6,860				
Insurance, debts, small sundry properties, personal goods, half cash at bank	35	980	No corresponding income.			
	£280	£7,840				

It is evident that the rates of interest shown in this table are higher than those in fact obtained. Indeed, in the Report of the Commissioners (Cd. 2633),* the net income from lands is stated at 4·3% (instead of 5·6%), and from buildings at 5·5% (instead of 8·7%). In the paper alluded to a searching examination is made of the reasons of this discrepancy, and it is found that in 1912–13 the taxable income arising from property is only 6·6% (instead of 7·6 as above) of the corresponding capital so estimated. This rate is still

* See also the table, pp. 80–1, in the 52nd Report (Cd. 4226).

higher than is in fact obtained, and till reconciliation is effected the total national capital cannot be safely estimated.

10. It is probable, however, that the increase of the total value of estates liable to duty observed over a period long enough to eliminate the accidents of individual years has a close relation to the growth of capital. The following table shows these values since the commencement of the duty.

Net capital value of estates which become liable to estate duty.		No. of millionaires included.
	£000,000's	
1895-96	213	8
1896-97	219	5
1897-98	247	7
1898-99	251	9
1899-00	293	12
1900-01	265	9
1901-02	289	8
1902-03	270	4
1903-04	264	7
1904-05	265	1
1905-06	272	8
1906-07	298	10
1907-08	282	7
1908-09	271	9
1909-10	284	5
1910-11	273	14
1911-12	278	5
1912-13	279	11
1913-14	296	11

In the first two years the totals are those of the capital on which duty was *paid*, which is less than the capital *liable* to duty which is that shown for the other years, since the payment is in some cases made in instalments.

CHAPTER X

TAXES AND RATES

1. The following table shows the "Amount of the Imperial Revenue (Exchequer Receipts, *net*) of the United Kingdom adjusted by deduction of charges in the year ended March 31, 1909." *

		0,000's		0,000's
Customs		£29,20		
Inland Revenue—				
Excise	£33,65			
Estate, etc., duties	18,37			
Stamps (exclusive of Fee and Patent Stamps)	7,77			
Land Tax	73			
House duty	1,90			
Property and Income tax	33,93			
Total Inland Revenue		96,35	Total revenue from taxes	£125,55
Post Office and Telegraph Service—				
Net receipts				4,81
Crown Lands		53		
Suez Canal Shares		1,06	Total from property	1,59
Total net revenue		£131,95		£131,95

* Detail, sufficient for most purposes, of revenue and expenditure is given in the *Statistical Abstract*; the Annual Finance Accounts of the United Kingdom (*e. g.* H. of C., 200 of 1909, price 5½*d.*) contain the more detailed official statement; the reports referred to in Chapter IX above give details of the Inland Revenue. The Commissioners of Customs and Excise also issue an Annual Report.

In the Exchequer receipts, unadjusted, there are also included £113,000 from interest or repayment of Sundry Loans (of which those to the Cunard Steamship Company and to the Gold Coast Colony account for £91,000), miscellaneous receipts amounting to £1,004,000, and receipts for Fee and Patent Stamps amounting to £1,023,000. These sums are received by, and redistributed from, the Treasury, but are treated as of the nature of cross-accounts, and do not enter into the balance-sheet of taxes and expenditure as generally understood.

2. The corresponding "Amount of the Expenditure (Exchequer Issues, *net*) of the United Kingdom, adjusted by deduction of extra receipts in the year ended March 31, 1909," is shown as follows:—

		0,000's
I. National Debt services		£31,39
II. Naval and military expenditure		56,78
III. Civil Services—		
(1) Civil List and Civil Administration—		
(*a*) Charged on Consolidated Fund	£1,38	
(*b*) Voted	10,69	
Less extra receipts	*1,76*	
Net		10,31
(2) Elementary education		15,44
(3) Old Age Pensions		2,07
(4) Charges transferred from local to imperial funds—		
(*a*) Charged on Consolidated Fund	9,82	
(*b*) Voted	3,59	
Less extra receipts	*6*	
Net		13,35
IV. Customs and Inland Revenue—		
Customs	97	
Inland Revenue	2,35	
		3,32
Total net Expenditure		£132,66
Total net Revenue		131,95
Excess of Expenditure over Revenue		71

As to I. The National Debt Services include a permanent annual charge applied to Interest, £19,000,000, and a sum

for repayment or reduction of debt (£9,000,000), and various smaller charges connected with temporary borrowings under various Acts.

As to II. The Naval and Military Expenditure is the total actually spent in the year, less sums charged to the National Debt. The account was—

	1907-8	1908-9
	£00,000's	£00,000's
Army expenditure	27,3	26,8
Navy expenditure	31,1	32,2
	58,4	59,0
Charges not met in the current year, but transferred, mainly to National Debt Services	2,3	2,2
Net expenditure . . .	£56,1	£56,8

There are also smaller items which complicate the account.

As to III. (1 *a*) The charge on the Consolidated Fund (£1,380,000) consists of the Civil List (charges for the expenses of the Crown), Annuities, Pensions, Salaries of Judges and other officials.

(1 *b*) The vote is for the annual expenses of the Administrative and Executive departments of the Civil Services.

(2) The vote for Elementary Education corresponds to the expenses of the Board of Education (£13,600,000), Public Education in Scotland (£2,000,000) and Ireland (£1,500,000), less payments in respect of Secondary Education. The residue, added to the sum paid from the rates (see p. 194 below) is the total public expense of elementary education including administration.

(3) This payment for old age pensions is for only a fraction of the year.

(4) Under this heading come the sums collected as taxes and used in reief of rates, or (what is nearly the same thing) for local purposes which would otherwise be paid for out

of rates. (*a*) The principal items here are £1,300,000 "Additional Beer and Spirit Duties" allotted since 1891 to Secondary or Higher Education, £4,400,000 allotted from the Estate, etc., Duties to the relief of rates generally and of agricultural rates in particular, and £3,000,000 received for licences and handed to the local authorities. (*b*) The votes account for many classes of payments, of which the largest items are: about £600,000 in lieu of rates on Government Property, about £1,100,000 for Prisons and Reformatories, and about £1,400,000 for the Irish Constabulary and Dublin Police.

As to IV. This heading covers the expense of collecting the taxes, and is a remarkably small percentage of their yield.

The details of, and relation between, the unadjusted and adjusted Revenue and Expenditure, of which only the latter are given here, can be studied in the Finance Accounts and in an explanatory memorandum called "Public Income and Expenditure" (*e.g.* H. of C., 258 of 1908, price 5*d.*).*

Corresponding figures for recent years are given on p. 189.

3. Turning to the details of Revenue, we find the receipts under Customs and Excise were as follows:—

	1894–5.	1902–3.	1908–9.
	£0,000's	£0,000's	£0,000's
Customs—			
Spirits	4,39	4,96	3,96
Sugar	—	4,48	3,16
Tea	3,59	6,00	6,05
Tobacco	10,42	12,45	13,82
Wine	1,14	1,52	1,12
Exported coal	—	1,99	—
Imported corn and grain	—	2,35	—
Others	80	93	1,05
Excise—			
Beer	10,49	13,71	12,69
Spirits	16,00	19,03	17,46
Licenses	3,75	4,26	3,11
Others	27	47	43
Total, Customs and Excise	50,85	72,14	62,85

* This memorandum was not published after 1908. See App. IV, p. 217, for later figures.

REVENUE AND EXPENDITURE, ADJUSTED AS IN PREVIOUS TABLES.

	1894–5.	1895–6.	1896–7.	1897–8.	1898–9.	1899–'00.	1900–1.	1901–2.	1902–3.	1903–4	1904–5.	1905–6.	1906–7.	1907–8.	1908–9.‡
	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000
Revenue—															
Customs and Excise	51,0	52,5	53,8	55,3	55,3	61,3	64,9	68,0	72,1	71,0	72,0	70,2	68,8	68,2	62,9
Property and Income-tax	15,6	16,1	16,6	17,3	18,0	18,8	26,9	34,8	38,8	30,8	31,3	31,4	31,6	32,4	33,9
Estate, etc., duties	10,9	14,0	14,0	15,3	15,6	18,5	17,2	18,5	18,1	17,2	16,7	17,3	19,1	19,1	18,4
Other taxes	8,2	9,8	9,8	10,1	10,0	11,0	10,3	10,3	10,7	10,1	10 4	10,9	10,6	10,7	10,4
Post Office	3,1	3,8	4,0	3,7	3,7	3,9	3,9	4.0	4,0	4,4	4,7	5,5	5,3	5,3	4,8
Crown lands and Suez Canal	8	1,1	1,1	1,1	1,1	1,3	1,3	1,3	1,4	1,4	1,5	1,6	1,6	1,7	1,6
Total net revenue	89,6	97,3	99,3	102,8	103,8	114,8	124,5	136,9	145,1	134,9	136,5	136,8	136,9	137,3	132,0
Expenditure—															
National Debt Services	25,0	25,1	25,2	25,3	25,2	23,5	20,4	22,5	28,4	28,8	28,9	30,9	31,6	33,2	31,4
Naval and Military	35,1	38,1	40,4	40,1	44,0	69,4	121,0	129,8	110,8	71,3	65,0	60,3	57,1	56,1	56,8
Civil List and Civil Administration	6,9	7,2	6,5	7,4	7,3	7,5	7,8	8,2	9,4	9,5	9,6	9,6	9,4	9,9	10,3
Elementary Education	9,0	9,3	9,8	10,6	11,0	11,2	11,6	11,8	12,2	13,3	14,2	15,0	15,4	15,7	15,4
Customs and Inland Revenue	2,6	2,7	2,7	2,7	2,8	2,8	2,8	3,0	3,0	3,1	3,1	3,1	3,2	3,2	3,3
In relief of rates	10,2	10,7	11,5	12,7	13,3	14,2	14,1	14,1	14,2	14,3	14,3	14,4	14,8	14,6	13,4
Old Age Pensions	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2,1
Total net expenditure	88,8	93,1	96,9*	99,1†	103,6	128,6	177,7	189,4	178,1	140,3	135,1	133,3	131,5	132,6	132,7

* Includes grant in aid to Egyptian Government, £800,000.
† Includes £250,000, expenses under the Coinage Acts of 1891 and 1893.
‡ For continuation see App. IV, p. 217.

The coal export duty was in force for six years, 1901–7; that on corn a little over a year; the sugar duty was introduced in 1901. In 1902–3 the total receipts were at their maximum for the period. In 1908–9 the various licences in respect of the manufacture and sale of alcoholic drinks amounted to about £2,200,000.

The estate duties, etc., were referred to in the preceding chapter. The principal items are Legacy Duty, Succession Duty, and Estate Duty. The last was introduced in 1894, in substitution for, and increase of, former duties, but the full yield was not realized immediately, while the year 1894–5 contains part of the old duties.

	1894–5.	1896–7.	1902–3.	1907–8.	1913–14.
	£0,000's	£0,000's	£0,000's	£0,000's	£0,000's
Legacy duty . .	2,81	2,55	3,00	3,91	4,48
Succession duty .	1,35	82	97	73	91
Estate duty . .	3,81	10,23	13,82	14,36	21,65
Miscellaneous .	2,92	28	12	11	13
	10,89	13,88	17,91	19,11	27,17

The receipts from Stamp Duties, Land Tax and Inhabited House Duty have varied little during the twenty years under consideration.

The Property and Income Tax was discussed in the last chapter. The tax was at 7*d.* in the £ in 1893–4, 8*d.* in 1894–99, 1*s.* in 1900–1, 1*s.* 2*d.* in 1901–2, 1*s.* 3*d.* in 1902–3, 11*d.* in 1903–4, 1*s.* in 1904–9, and 1*s.* 2*d.* in 1909–10. Exemptions and abatements were extended in 1894 and 1898.

4. The Inhabited House Duty is specially interesting for the statistician, for in the tables relating to it (52nd Report of the Commissioners of the Inland Revenue, pp. 113 *sqq.*) we have information as to the assessed value of all the inhabited houses and residential shops and premises, and in less detail of uninhabited premises, in England, Wales and Scotland

The duty is not imposed in Ireland. The following tables show the nature of the information :—

GREAT BRITAIN, 1907-8.*

Exempt from duty.	No. of premises.		Annual value.	
			£000's	
Premises not used as dwellings	664,266		49,819	
† Separate dwellings exempt from duty	64,681		845	
Royal and diplomatic residences, hospitals, schools, etc. . .	33,872		4,089	
Houses of annual value— under £10 . .	3,162,752	6,112,736	20,130	59,966
£10 and under 15 . .	1,985,639		23,463	
15 and under 20 . .	964,345		16,373	

Charged to duty.	No. of premises.		Annual value.	
	Private dwelling-houses.	Others.‡	Private dwelling-houses.	Others.‡
			£000's	£000's
†"Separate dwellings"—				
£20 and under £41 .	19,261	—	486	—
41 " 61 .	4,695	—	234	—
Houses—				
£20 and under £25 .	369,640	85,219	8,069	1,820
25 " 30 .	248,531	65,183	6,595	1,708
30 " 41 .	402,454	123,662	13,818	4,248
41 " 50 .	103,352	34,115	4,600	1,532
50 " 61 .	123,072	50,144	6,646	2,735
61 " 80 .	61,151	29,821	4,195	2,079
80 " 100 .	38,245	20,919	3,300	1,812
100 " 150 .	44,581	22,435	5,227	2,625
150 " 200 .	16,468	9,154	2,733	1,514
200 " 300 .	13,460	7,138	3,137	1,670
300 " 400 .	5,199	3,122	1,725	1,040
400 " 500 .	2,370	1,531	1,024	664
500 " 1000 .	2,826	2,328	1,827	1,507
1000 and over . .	970	836	2,093	1,870
	1,456,275	455,607	£65,710	£26,825

* The statistics are subject to slight additions when arrears are collected.
† That is, parts of buildings (*e. g.* flats) used as separate dwellings.
‡ Residential shops, hotels, public-houses, etc., farmhouses, lodging-houses.

In a different form the statistics show :—

	Nos. of private dwelling-houses, whether charged to or exempt from duty.			
	Metropolis.	Rest of England.	Scotland.	Great Britain.
	000's	000's	000's	000's
"Separate dwellings" exempt	54	11	—	65
" " £20 to £61	21	3	—	24
Houses to £10 . .	7	2,580	575	3,163
£10 " 15 . .	42	1,765	178	1,986
15 " 20 . .	73	803	89	964
20 " 25 . .	92	248	30	370
25 " 30 . .	56	169	24	249
30 " 41 . .	125	243	34	402
41 " 100 . .	108	188	29	326
100 " 500 . .	27	49	6	82
500 . . .	3	1	—	4
	608	6,060	965	7,634

The importance of these statistics is in their relation to the social grading of the people, a subject with which the population census does not deal, and also in relation to the statistics of income. The income-tax returns and the value of houses cannot easily be compared, but there is here a possible field of investigation of a difficult character. In general (but with many exceptions) there is one private dwelling-house to one payer of income-tax, and also in general (but with some extraordinary exceptions) the higher the income the larger the value of the house occupied, but the smaller the proportion of income spent on rent; this proportion probably varies from 25% for some classes of workmen in the large towns to 10% for persons with an income of £700 a year. The number of income-tax payers in Great Britain is probably a little less than the aggregate number of houses of value above £30 in London and above £25 in the rest of Great Britain. The aggregate annual value of these houses was about £55,000,000 in 1907; the aggregate income of income-tax payers in Great Britain was somewhat over £600,000,000. There is much that is hypothetical in this comparison, but it suggests an interesting line

of analysis. An inquiry by sample as to the average income of the inhabitants of different types of houses would lead to an independent estimate of the aggregate of individual incomes in Great Britain.

5. From the table of expenditure on p. 186 above, and the explanation under III, p. 187, it is clear that Local and Central Expenditure cannot be separated from each other; and though rates and taxes are generally paid to different authorities, they are equally a compulsory drain on the pockets of the payer. We will, therefore, investigate the total sum expended locally in the year 1911-12, the last for which the figures are available. The tables in the Statistical Abstract on Local Finance need careful interpretation.

LOCAL AUTHORITIES.

Receipts, 1911-12.	England and Wales.	Scotland.	Ireland.	United Kingdom.
	£00,000's	£00,000's	£00,000	£00,000's
Rates	66,4	7,0	3,3	76,7
Government contributions.	22,3	3,0	1,4	26,7
Tolls, dues, and duties .	8,0	1,3	0,4	9,7
Rents, interest, sales of property	4,3	0,4	0,3	5,0
Fees, fines, etc. . . .	1,5	0,2	0,1	1,8
Water, gas, electric light, tramways, and light railways	27,3	5,2	1,2	33,7
From loans	17,7	2,4	1,2	21,3
Miscellaneous . . .	5,2	0,6	0,4	6,2
Repayments for improvements	1,3	—	—	1,3
	£154,0	£20,1	£8,3	£182,4

It is not practicable without a long investigation to allot the whole of this £182,400,000 to categories of expenditure, and there are innumerable cross-accounts with the Central Government, with Capital and Interest balances, with municipal trading undertakings, and with the allotment of particular receipts to particular purposes. The table is given mainly for the purpose of exhibiting the difference between the sum drawn in rates and total receipts.

6. It is not at all easy to estimate the annual expenditure in direct and indirect taxation and in rates, the receipts of which are applied to public purposes, national or local, and to separate it from receipts from and payments for loans, interests and profits. The following table, however, gives a

PUBLIC RECEIPTS AND EXPENDITURE.

	1891–5.	1905–6.	1911–12.
	£000,000's	£000,000's	£000,000's
NATIONAL RECEIPTS.			
Indirect taxation—			
Customs, excise and stamps .	57	78·5	81·5
Direct taxation—			
House duty, land tax, income tax, and estate duties . . .	29	51·5	73·5
Total from taxes . . .	86	130	155
Profits from post office * . .	3	5·5	5·2
Crown lands and Suez Canal .	1	1·5	1·8
Total national receipts . . .	90	137	162
LOCAL RECEIPTS.			
Rates	40	67	77
Total receipts . . .	130	204	239
EXPENDITURE—			
National debt†, army, navy, civil list, and civil services‡, excluding education	71	107	122
Insurance, labour exchanges, old age pensions	—	—	12
Education § (including administration), whether paid from taxes or rates	18	36	105 ¶
Poor relief ‖	12	17	
Other local purposes met directly out of rates and taxes . . .	29	44	
Total expenditure . .	130	204	239

* This may be regarded either as indirect taxation or as trading profits.
† Interest and repayment, allotting the excess of income over expenditure shown on p. 189 above to repayment of debt.
‡ Including expenses of customs and inland revenue.
§ Including secondary education, colleges and museums. Of course, large sums are privately subscribed and spent on education, as well as the amounts here given.
‖ "Expenditure by unions and parishes in relief of the poor."
¶ This figure should be increased by 3, 4 or 5 to make it comparable with previous years, and the 122 above diminished owing to cross-reckonings explained in App. IV, p. 217, note b.

rough idea of the public contributions and the main objects of expenditure.

The figures in the line "Other Local Purposes" are obtained simply by subtraction of the expenditure detailed in the previous lines from the total receipts.

From the data given on p. 172 above, it appears that as much as $\frac{1}{8}$th of the Aggregate of Individual Incomes is taken in rates and taxes, direct or indirect.

From 1894 to 1911 total income probably increased about 35 %, rates and taxes together about 84 %. A very rough idea of the change since 1860 may be given as follows. Rates and taxes as a percentage of total income: 1860, 12 %; 1880, 9 %; 1890, 8 %; 1894, 9 %; 1905, 11 %; 1911, 12 %. It is certainly probable that the percentage fell from 1860 to 1890, and the rough figures indicate that the grant of old age pensions and the expenses of the Insurance Act will have by 1914 brought the proportion at least up to that of 1860. These statistics rest on uncertain estimates, and should only be used for very rough purposes.

EQUIVALENT TABLES IN STATISTICAL ABSTRACTS

	1914	1917	1920
Table Number	16	16	18
" "	22	22	Cancelled
" "	24	24	25
" "	33	Cancelled	Cancelled
" "	34	33	32
" "	35	34	33
" "	40	38	36
" "	41	39	37
" "	48	45	43
" "	49	46	44
" "	52	48	46
" "	55	49	47
" "	56	50	48
" "	60	54	52
" "	61	55	53
" "	69	63	61
" "	70	64	62
" "	75	69	69
" "	77	71	71
" "	78	72	72
" "	81	75	66
" "	82	76	67
" "	84	78	75
" "	117	110	105
" "	118	111	106
" "	119	112	107
" "	150	143	136
" "	151	144	137
Page	400	376	316
"	432	404	340

APPENDIX I

PART I

Exercises on Chapter II

[References are to tables in the preceding pages, or to the *Statistical Abstract for the United Kingdom*, 1914, Cd. 7636, price 1*s.* 11*d.* If the Abstract for 1917 (Cd. 491) or for 1920 (Cmd. 1774) is used, alterations must be made as shown on the page opposite.

Where dates are given which do not appear in the tables of the recent abstracts, substitute other years at discretion.

1. Write down the number of bushels (p. 6) in the forms (*a*) to (*f*).

2. Add together 75,324, 79,476, 432,132, the numbers being correct to 1 %, 2 %, 3 % respectively.

3. The population of a colony consists of 73,243 Europeans, $7{,}8^{00}$ Indians and 432^{000} negroes. What is the whole population?

4. The average wage of $2{,}456^{000}$ workmen is 26*s.* 6*d.* (to nearest 6*d.*). What is their aggregate wage?

5. The total income of $3{,}254{,}6^{00}$ persons is $£243 \times 10^6$ What is the average income?

6. The productivity of $4{,}325^{000}$ acres is between 38 and 39 quarters of wheat per acre. A quarter weighs between 470 and 490 lbs. What is the yield in tons?

7. £87,547 is to be raised in rates, where the assessed annual value is £943,650. Find the least rate nêcessary (fractions of one farthing not being used) and the excess collected.

8. The quantity of wheat imported in 1894 was $70{,}126^{000}$

cwts., in 1908 $91,131^{000}$ cwts. Express the ratio in the notations of pp. 11, 12.

9. If wages per hour rose 20 %, and the number of hours worked per week fell 10 %, find the change in weekly wages.

10. Wages were raised 10 %, lowered 15 %, raised 20 %, lowered 25 %, and raised 10 % in certain years, each percentage being reckoned on the wages current when the change was made. Find the change in the whole period.

11. Express the 4 summary totals of the value of imports (*Stat. Abs.*, end of Table 41, p. 167) in 1900, 1908, 1913 as percentages of the grand total.

12. In the same table express the totals A to O under III (Articles Wholly or Mainly Manufactured) as percentages of the total of III in 1913.

[For further treatment of Approximations and the Contracted Method, and numerous examples, the student is referred to *Arithmetic, Mainly Examples*, G. W. Palmer, Macmillan & Co.]

Exercises on Chapter III

1. Apply the monthly average prices of wheat in 1913 (*Stat. Abs.*, Table 75, p. 331) to the quantities (on p. 333), to find the total values sold as accurately as the data allow.

2. Find the average production per acre of the 6 corn crops shown in Tables 77 and 78 for (1) Great Britain, (2) Ireland, for the years 1899 and 1913.

		Acreage.	Average yield per acre.
3.	A.	3,456,789	35·2 bushels
	B.	2,703,257	30·7 "
	C.	1,432,843	43·8 "

Find the average yield in A, B, C together by the methods of p. 18.

4. Using the methods of p. 20, check the averages shown in the tables in Part I, Ch. VI, p. 51, and in Part II, Ch. VI, p. 149, suggesting the cause of the discrepancies (if any) found.

5. Find the arithmetic average, the median, the quartiles and the mode of the miners' ages given in Part I, Ch. V, p. 36.

6. The prices of 4 kinds of wool are calculated in the *Stat. Abs.* (Table 60, p. 288), from data given in Tables 40 and 41. Check the calculations for the year 1908, and find the average price for the four kinds lumped together. Observing that the data are incomplete in 1900, consider how accurately an average for 1900 (comparable with those of subsequent years) can be computed.

[Observe that camel's hair is 2*d.* to 2½*d.* per lb. cheaper than sheep's wool, and that the maximum import is less than that shown under K, p. 138.]

7. Criticize the following averages: Table 24, p. 55. Total spent in poor relief, 1905–6, England and Wales: £11,675,959. Total relieved (p. 432): July 1st, 1905, 863,523; Jan. 1st, 1906, 919,751. Average number relieved, 891,637. Average cost, £13 1*s.* 11¼*d.*

8. If the average wage of 55,000 men is 28*s.* 6*d.*, and of these the average for 30,000 is 25*s.*, find the average for the remainder.

9. Find d_1, d_9 (the 1st and 9th "deciles"), in the table on p. 23, so that "one-tenth of the wage-earners received d_1/- or less, and one-tenth received d_9/- or more."

Exercises on Chapter IV

1. Write the net value of Consignments (*Stat. Abs.*, Table 34, pp. 80–81) from the 28 British Possessions for 1904 and 1913 in 6 columns as in the table in Part I, Ch. IV, p. 27, and calculate the ratios of the totals. Make

two new columns also showing the values to the nearest million £, and calculate the ratios.

Find also the relative and the absolute errors in the totals of each of the columns and comment on the results.

2. Write the table of monthly prices (*Stat. Abs.*, Table 75, pp. 330, 331) for the years 1900, 1908, 1913, (1) omitting pence, (2) to the nearest shilling. Find the averages for the years, and express them as percentages of the average found for 1900. Comment on the result.

3. Re-write the table showing the percentage of unemployed monthly from 1897 to 1906 (Part II, Ch. VII, p. 158), (1) omitting the decimals, (2) to the nearest whole numbers. Calculate the yearly means and the 10 years, monthly averages. Comment on the result.

4. Find the total yardage and total value of woollen tissues exported in 1908 from *Stat. Abs.*, Tables 48 and 49. Calculate the average price.

Now write down the average prices of the eight qualities enumerated from Table 61. "Weight" these averages (1) with the number of million yards of each kind, (2) with the values to the nearest £100,000, and find the "weighted average" price. Explain why the result of (1) agrees very closely with average already calculated, while the result of (2) is 40% too large.

Exercises on Chapter V

1. Make diagrams of the type on p. 37 of the wages shown in Part II, Ch. VI, p. 149.

2. Make circular diagrams (as on p. 47) of the main items of revenue and expenditure in the first and last years shown in Part II, Ch. X, p. 189.

3. From the Tables of Imports and Exports (*Stat. Abs.*, Table 33 or 34) make the following diagrams, for the years 1899–1913:—

(i) Of Imports from Foreign Countries, British Possessions and Total.
(ii) Of Exports to Foreign Countries, British Possessions and Total.
(iii) Total Imports and Exports.
(iv) Imports and Exports to and from British Possessions.

4. From Tables 48 and 49 (*Stat. Abs.*, pp. 196-7, 214-15) make three diagrams of the value and quantity of Cotton Piece Goods exported, (1) representing 100 yds. and £1 by same unit, (2) making the lines start together, (3) making the lines end together.

5. Represent the numbers in the table on p. 180, Part II, Ch. IX, by a diagram.

6. Treat one or more of the columns of the prices in the table on p. 128, Part II, Ch. IV, by the method of p. 42.

Exercises on Chapter VI

[Use round numbers throughout and pay attention to clearness of meaning and legibility.]

1. Make a table of Imports and Exports (*Stat. Abs.*, Table 33), grouping together the Foreign Countries in Europe, Asia, Africa and America, and showing British Possessions in 5 groups, from 1899 to 1913.

2. Make a similar table of Consignments from various groups of countries, 1904 to 1913 (*Stat. Abs.*, Table 34).

3. Make a table comparing Imports from and Consignments from those countries in which the values are markedly different.*

4. Make a series of brief tables showing the information as to the Woollen and Worsted industry contained in the *Statistical Abstract.*

5. Make a table combining imports and exports of bullion

* This exercise cannot be done if an Abstract more recent than that for 1914 is used.

and specie with those of merchandise (*Stat. Abs.*, Tables 34, 35, 55 and 56) for each of the principal gold or silver producing countries.

Exercises on Chapter VII

1. Find by sampling the number of words (1) in a full line of this book, (2) in lines including those at the beginning and end of a paragraph. Hence estimate the number of words in a page containing 37 lines. Calculate the precision of your estimate, and verify it by counting the number of words in a number of pages.

2. Make a similar estimate for the average number of letters in a word. Also by taking, say, 1000 words, find the frequency of words of different lengths. Estimate the precision of your results, and verify it by tabulating a large number of consecutive words.

3. Find the ratio of the number of commas to the number of full stops in this or any other book.

4. Find whether the digits 0 to 9 are uniformly distributed through the table at the beginning of Chap IV, Part I.

Exercises on Chapter VIII

1. Apply the rules of criticism given to—

   (1) the various statistics relating to Paupers in Table 150, 151, pp. 432–3, *Stat. Abs.* (England and Wales).
   (2) The Post Office statistics, pp. 400 *sqq.*
   (3) the categories of expenditure by Local Authorities in the United Kingdom, Table 22, p. 52 with reference also to Table 24.
   (4) The Income Tax categories, Table 16.

2. How far can the statistics of (1) wages, (2) consumption of meat, (3) value of exported manufacturers, be regarded as tests of National Progress?

3. Which of the average prices, Table 60 (*Stat. Abs.*), satisfy least the criteria of paragraph 5?

4. Do the tables relating to railways, Tables 81, 82 (*Stat. Abs.*), give the information that statisticians want?

Exercises on Chapter IX

1. Verify the averages in the table in paragraph 3. Deduce the number of miles of track and of route. Show that ton-miles per engine-hour, divided by wagon-miles per engine-hour would equal the average full-and-empty wagon load; and that wagon-miles per engine-hour, divided by train-load would give train-miles per engine- (train and shunting) hour.

2. Consider what data would give the best information for any business or institution with which you are acquainted.

3. Make a blank card suitable for entering details as to a workman applying at a Labour Exchange.

4. Draw up a blank schedule suitable for tabulating details of working-class expenditure.

5. Required to describe the housing accommodation of a district. How would you proceed and what blank forms would you use (1) if you had legal power of entry and measurement, (2) if the inquiry was on a voluntary basis?

PART II

Exercises on Chapters I and II

1. Calculate some of the *birth-rates* in Table 119 (*Stat. Abs.*, p. 408), from the number of births and from the population stated in the preceding table.

2. Estimate the population of Scotland for each year from 1891 to 1901, using only the data of Table 117 (p. 407), and compare your results with those of Table 118.

3. Work out from the Census Report for your county the density of population in as much detail as possible in your neighbourhood.

4. From the statistics of population, births and deaths (Tables 117, 118, 119), find the excess of emigrants over immigrants for Scotland and for Ireland between 1901 and 1911.

5. With the help of a diagram estimate the actual and relative number of men between the ages 32 and 38 in table on p. 98 above; and also the number of children between 7 and 14. (The whole population is given on p. 89.)

6. Find the actual numbers in various occupations from the per mille table on p. 91, and state in what cases the absolute numbers have increased while the relative numbers have diminished.

7. How is it that the infant mortality rate is lower than the death-rate between 0 and 1 years?

8. In Table 119 (*Stat. Abs.*, p. 408) calculate the population in 1911 from the number of births and the birth-rate, as accurately as these data allow, and compare with the two preceding tables.

9. Find the corrected death-rate for District B to compare with District A as standard, by both the methods described on pages 105, 106 from the following data. Find also the general uncorrected death-rates.

		Years 0-5.	Years 5-15.	Years 15-55.	Years 55-
District A.	Relative number of persons .	114	110	670	106
	Death-rates	4	3	7	60
B.	Relative number of persons .	136	125	619	120
	Death-rates	38	3	6	55

Exercises on Chapter III

1. Make the tables corresponding to those in paragraphs 3 and 4 for the years 1900 to 1908.

2. Make a table of the excess of imports over exports, (including bullion) for the years 1899-1913, and express this excess year by year as a percentage of the total of imports and exports. On the same diagram show the numbers in this table, and the total tonnage of vessels registered as belonging to the United Kingdom (*Stat. Abs.*, Table 70, p. 326).

3. Draw diagrams showing (1) total value of imports and total tonnage of ships entered with cargoes, and (2) total value of exports and total tonnage of ships cleared with cargoes (*Stat. Abs.*, Table 69, pp. 324–5).

4. Draw smoothed diagrams (as Diagram III, p. 42, above) representing the table of external trade on p. 118 above.

5. Illustrate the process described in paragraph 8 by valuing the quantities of imported meat 1899 to 1913 (*Stat. Abs.*, Table 40, pp. 132, 133) at the prices of 1899 (Table 60, p. 288), and comparing the totals obtained with the declared values (Table 41, pp. 148, 149). Show the result on a diagram. (See also Exercise 1 on Chapter IV.) [Work only to three significant figures.]

6. Express the following categories of exports: Food, drink and tobacco—Coal—Other unmanufactured commodities—Iron and steel manufactures—Cotton and wool products—Other manufactured commodities—as percentages of total exports of home produce, year by year from 1899 to 1913. [Use either the table in paragraph 9, or the *Stat. Abs.*]

7. Draw a diagram illustrating the increase of steamships relative to sailing from *Stat. Abs.*, Table 69, pp. 324–5.

Exercises on Chapter IV

1. Make index-numbers of the prices of imported meat from the figures obtained in Exercise 5 on last chapter.

2. Calculate index-numbers for 1880, 1890, 1900, 1910 and 1913, for the eight commodities together (wheat to coal) shown in paragraph 4 (1) taking 1865–9 as the basis, (2) taking 1875 as the basis, (3) taking 1900 as the basis. In each case re-write the index-numbers so that the number for 1913 is 100. Comment on the differences shown.

[Note.—So few commodities are, of course, insufficient for establishing a general index-number.]

3. Transfer Sauerbeck's index-numbers from gold values (in which they are given on p. 128) to silver values.

4. From the *Stat. Abs.* (Tables 60, 61, 84) make a table and diagram comparing the prices of pig-iron imported, exported and produced.

Exercises on Chapter V

1. Make a table for 1900–1913 from the *Stat. Abs.* showing the value of imported raw cotton (less re-exports) as compared with the value of exported cotton goods. Assuming that 80 % of imported cotton is used for the foreign trade, find the value added by manufacture year by year.

2. The Census of Production shows that the value of the Output of Cotton Factories in 1907 was £45,000,000 more than that of cost of materials used. The value of cotton imported and retained that year was £61,000,000, and of exported cotton manufactures was £110,000,000. If these statements are consistent with the 80 % assumption of the last exercise, deduce the value of materials used (coal, etc.) other than raw cotton.

3. Tabulate the values of the home production, imports and exports, of iron, steel and their products for 1908 from Tables 41, 49, 52, and 84 of the *Stat. Abs.*

EXERCISES ON CHAPTER VI

1. The wages of 2,000 men were decreased $\frac{1}{2}d.$ per hour and the normal week was increased 3 hours. If before the change the rate was 10*d.* and the week 50 hours, compute the effect that would be shown in a "change of wages" table.

2. If average weekly wages in Textiles, Agriculture, Building, and Engineering had been respectively 15*s.*, 13*s.*, 25*s.* and 27*s.*; and the relative numbers employed 5, 10, 2 and 3 in 1880, compute the change per cent. for the 4 groups together in 1890, 1900 and 1908 from the index-numbers in paragraph 9, (1) assuming no change in the relative numbers, (2) assuming that the numbers changed gradually till in 1908 they were 5, 7, 3, 5.

3. If average wages rise 20 %, and the retail purchasing power of money rises 10 %, how much do average real wages rise?

4.

Wages Grade.	Number of men. Year 1.	Year 2.
26*s.*–28*s.*	25	15
28*s.*–30*s.*	25	25
30*s.*–32*s.*	25	35
32*s.*–34*s.*	25	25

Find the maximum and minimum change possible in average wages consistent with promotions as shown in this table, assuming that no man's wage was reduced.

5. Compute the lines for lads and girls on page 149 on the assumption that all receiving less than 5*s.* were half-timers (none earning less than 2*s.* 6*d.*) and supposing each

pair of half-timers replaced by one full-timer at their joint wages.

Exercises on Chapter VII

1. Make diagrams illustrating the table on p. 156.

2. For columns A and B of the same table take decennial averages for 50 periods beginning 1851, 1852, to 1900, and represent the result in a diagram. Comment on the result.

3. Compute column D counting B_2 as twice as important as B_1.

4. Which of the lines in the table on p. 157 can be calculated from the other lines?

5. Make a diagram showing the general percentage of unemployment, as shown in paragraphs 6 and 7, pp. 158–160, for every month from January 1897 to October 1909.

If seasonal changes are eliminated, which was the worst month in 1904–5?

Exercises on Chapter VIII

1. Express the expenditures shown in the table on p. 164 as percentages of the total expenditure.

2. What information does the *Stat. Abs.* contain as to working-class savings?

3. Check the percentages shown on pp. 168, 169.

4. How far can the higher expenditure shown (pp. 168–9) for English incomes than for French or German over 40*s.* be accounted for by the larger number of children?

Exercises on Chapter IX

1. If the income-tax is 1*s.* 2*d.* on unearned incomes and 9*d.* on earned when the total is less than £2,000, and abate-

ments are as shown on p. 175 and are made on the earned portion of income, find the income-tax payable and the actual rate in the £ in the following cases—

	Earned.	Unearned.
	£	£
Income of A	300	10
B	400	100
C	300	200
D	501	200
E	495	200

		£
2. The net profits of a firm were in	1900	5,500
	1	5,000
	2	4,500
	3	4,400
	4	4,000
	5	4,000
	6	5,000
	7	6,500
	8	6,000

How would these appear in the returns of income if the average for the three preceding years were taken yearly as described on p. 178.

Exercises on Chapter X

1. If the whole of indirect taxation were borne by working-class families and others with incomes below £160, and the whole of direct taxation by income-tax payers, and if the two classes consisted respectively of 7,000,000, and 1,000,000 families and their aggregate incomes of £1,000 mln. and £800 mln., calculate the burden per family in each case and the proportion of taxes to income in each case. [Use the Table, p. 194, omitting Office and Crown Lands, etc.]

2. Estimate the aggregate income of Great Britain on the hypothesis that among persons where the rent is less than £25, the average family income is 8 times the rent

£25 to £50	"	"	10	"
£50 to £80	"	"	12	"
£80 to £500	"	"	15	"
£500 and over	"	"	20	"

Miscellaneous Examination Questions

1. In the following table the density is measured by the number of persons to the square mile and the population in each line is given correct to the nearest thousand:

Density of districts.	Inhabitants. 000's
50–100	15
100–200	50
200–300	55
300–400	40
400–500	25
500–600	10
600–700	5

Estimate the area of the aggregate of the districts and also the density of the aggregate. What is the maximum density that is consistent with the data?

2. 5,300 children under 15 years old form 31 % of a population. If the number of children is given to the nearest 100 and the percentage to the nearest unit, to what degree of accuracy can the population be estimated from this statement?

3. Of 743,000 cwt. the average price of 21 % was 58s. and of the rest 74s. per cwt. If the price is stated to the nearest shilling and the quantity to the nearest 1,000 cwt., find

the greatest and least amounts that the whole can have cost.

4. 465,000 persons were employed in coal-mines in November 1918, whose average earnings were £13 in four weeks. Of these, 139,000 were coal-getters, with average earnings £15 10*s*. Calculate the average earnings of other operatives, supposing the figures exact, and find also the minimum wage consistent with the data, if the numbers are given only as the nearest thousand and the earnings as the nearest 10*s*.

5. The average wages of two groups, containing n_1 and n_2 persons respectively, are a_1 and a_2, and the average of the two groups merged is A. $n = n_1 + n_2$. A, n, n_1 and a_1 are known approximately, but each may be 1 % in error in excess or defect. A is less than a_1.

Show that the greatest value of a_2, consistent with this statement, is obtained when A and n are taken as great, and a_1 and n_1 as small as possible, and work out the result when $A = 40$, $a_1 = 45$, $n = 73{,}700$, $n_1 = 30{,}600$.

6.

Occupied in Hat Manufacture

	All.	On Piece-rates.	On Time-rates.
	%.	%.	%.
Males . . .	63	53	79
Females . . .	37	47	21
	100	100	100

From this table find the percentages paid piece-rates (1) of all employed, (2) of all males, and (3) of all females.

7. If, in an industry employing men and women, men form 40 % of all employed, and men paid time and piece-rates form respectively 20 % and 45 % of all paid time and piece-rates, find the proportion of all employed, and also of men and of women employed, who are paid piece-rates.

8.

COTTON TRADE

Number of workpeople.		Total wages.	
August 1922.	Increase over August 1921.	August 1922.	Decrease from August 1921.
89,026	6·5 %	£168,505	6·6 %

Compute the numbers and wages in August 1921 and the change in the average wage.

9. Sauerbeck's index-numbers for 45 commodities in 1916 (the averages of 1867–77 being taken as 100) were 107, 121, 114, 132, 128, 163, 131, 168, 138, 154, 148, 157, 169, 148, 153, 100, 93, 84, 68, 135, 166, 154, 173, 159, 125, 197, 100, 104, 172, 161, 163, 159, 101, 71, 174, 160, 104, 114, 119, 135, 96, 86, 128, 183, 202 respectively.

Find the arithmetic mean, median, quartiles and quartile deviation of these numbers.

10. Find the average, median, mode and one measurement of deviation in the following frequency table:

AVERAGE SIZE OF FAMILY IN 128 DISTRICTS

Persons per family.	Number of districts.	Persons per family.	Number of districts.
3·5 to 3·6	1	4·3 to 4·4	32
3·6 ,, 3·7	1	4·4 ,, 4·5	23
3·7 ,, 3·8	2	4·5 ,, 4·6	10
3·8 ,, 3·9	1	4·6 ,, 4·7	11
3·9 ,, 4·0	2	4·7 ,, 4·8	2
4·0 ,, 4·1	3	4·8 ,, 4·9	0
4·1 ,, 4·2	8	4·9 ,, 5·0	1
4·2 ,, 4·3	30	5·0 ,, 5·1	1

11. A weighted average, Q, is obtained by applying approximate weights $w_1, w_2 \ldots$ to known quantities $q_1, q_2 \ldots$ Show that, if the weights are slightly modified so as to be $w_1 + e_1, w_2 + e_2 \ldots$, Q becomes $Q + S(q_1 - Q)e_1/Sw$ approximately.

12.

IMPORTS INTO THE UNITED KINGDOM

	Unit.	Quantities. 1913.	1918. (000,000's.)	1919.	Values. 1913.	1918. (£000,000's.)	1919.
Wheat	cwt.	106	58	71	44	53	68
Beef	cwt.	9	8	6	16	36	30
Cotton	lb.	2,174	1,489	1,958	79	150	190
Wool	lb.	801	413	1,043	34	36	97

Show the change in volume of the total importation of these four commodities when weights are based on (*a*) 1919 values, (*b*) 1913 values.

13. Point out the ambiguities or errors in the following statements, and if possible word them correctly:

(1) The following table shows the increase of the price of vegetables in Germany (*Metalarbeiter-Zeitung*):

	1914. (Pfennigs.)	Nov. 1917. (Pfennigs.)	Per cent. increase.
Potatoes	2·5	8	220·00
Carrots	3	12	300·00
Kohlrabi	1·5	8	433·33
White cabbage	3	25	733·33
Onions	6	25	316·66
Average increase			400·66

(2) "The increase in prices in Stockholm was 110 % from July 1914 to December 1917; in the first year it was 26 %, in the second a further 12, in the third year (up to July) 35, and during the last half-year 35."

(3) The average size of a family decreased from 4·5 to 3·5 in 10 years, and this explains the slower growth of the population.

(4) While rates of wages increased 10 %, income subject to tax increased 15 %; hence wage-earners are losing relatively.

(5) In the two periods 1902–6 and 1906–12 the purchasing power of the sovereign fell equally, for the index-numbers of prices were 69, 77, and 85 in 1902, 1906, and 1912 respectively.

(6) New-laid eggs were sold at 6 to the shilling, imported at 10 to the shilling. The average was therefore 8 to the shilling.

(7) In 1901 and 1911 the populations of New Zealand were 20·0 and 22·6 % of those of Australia, a relative increase in 10 years of 11·30 %.

(8) In one period, money wages rose 10 % and prices fell 5 %, and in the next wages fell 8 % and prices rose 7 %, so that in all real wages neither rose nor fell.

(9) It is observed that married men live longer than unmarried; hence we conclude that marriage is conducive to health.

(10) The wages of each group rose 5 % and therefore the wages of the aggregate rose 5 %.

APPENDIX II

Selected List of Books of Reference

Jevons—Investigations in Currency and Finance. Macmillan.
Giffen—Economic Inquiries and Studies. G. Bell & Sons.
Goschen—Essays and Addresses on Economic Questions. Arnold.
Bowley—Official Statistics. Oxford University Press.
Webb—New Dictionary of Statistics. Routledge.
The Journal of the Royal Statistical Society. 30/- annually.

Government Publications.

Government publications are either issued by one of the Ministries or Departments without a code number, or as "Command Papers," or "House of Commons Reports or Papers." In the first case it is necessary to quote the abridged title of the report and the date, when ordering; in the second case the code number is sufficient, *e. g.* Cmd. 1774 (prior to 1900 C. was the prefix, from 1900 to the end of 1918 Cd. and subsequently Cmd.); in the third case such an abbreviation as H. C. 365 of 1906 is sufficient.

Periodical Publications.

The reference number is for the last issued prior to February 1923.

	Number.	Price.	
Statistical Abstract for the United Kingdom	Cmd. 1774	10	0
,, ,, ,, Principal Foreign Countries	Cd. 7525	1	9
,, ,, ,, British Oversea Dominions	Cmd. 1630	12	6
,, ,, ,, British India	Cmd. 1778	6	0
Abstract of Labour Statistics	Cd. 7733	1	6
Trade of the United Kingdom in 1921, Vol. I		12	0
,, ,, ,, ,, ,, 1920, Vol. II	Cmd. 1504	40	0
,, ,, ,, ,, ,, 1920, Vol. III	Cmd. 1505	27	6
,, ,, ,, ,, ,, 1920, Vol. IV	Cmd. 1506	30	0
Monthly Trade & Navigation Accounts. The twelve Months ending December 1922	H. C. 26 viii.	6	0
Quarterly Trade & Navigation Accounts for Foreign Countries, etc.	H. C. 27 iii.	3	0
Navigation and Shipping in 1920	Cmd. 1442	17	6
League of Nations Monthly Bulletin		1	6
Ministry of Labour Gazette. Monthly[1]			6
Registrar General's Report on Births, etc., for 1920[2]		20	0
Supplement to Registrar General's Report, Part I (issued decennially)	Cd. 7512		
,, ,, ,, ,, ,, Part II	Cmd. 1010	1	6
,, ,, ,, ,, ,, Part III	Cd. 8002	8	0
Ministry of Health. Annual Report for 1921-22.	Cmd. 1713	6	0

[1] Better known as the *Labour Gazette.*
[2] To be replaced by The Registrar General's Statistical Review for 1921, and subsequent years, in three parts.

	Number.	Price.	
Mines and Quarries. General Report, 1920, Part I.			3
,, ,, ,, ,, ,, Part II.		2	0
,, ,, ,, ,, ,, Part III.		1	6
Agricultural Statistics, Vol. LVI, Part I, 1921 .		2	0
,, ,, ,, Part II, 1921 .		1	0
,, ,, ,, Part III, 1921 .		1	6
Railways: Capital Traffic, etc., 1921		21	0
Railway Statistics—Great Britain. Monthly .		2	6
Finance Accounts, No. 88		2	0
Reports of Commissioners of the Inland Revenue, 1913–14	Cd. 7572	1	6
Reports of Commissioners of the Inland Revenue, 1920–21	Cmd. 1436	4	0
Reports of Commissioners of Customs and Excise, 1920–21	Cmd. 1435	5	0

Special Reports.

	Year.	Number.	Price.	
Statistical Tables and Charts relating to British and Foreign Trade and Industry . .	1903	Cd. 1761	3	6
Statistical Tables and Charts relating to British and Foreign Trade and Industry . .	1904	Cd. 2337	3	6
Statistical Tables and Charts relating to British and Foreign Trade and Industry . .	1909	Cd. 4954	5	2
Public Health and Social Conditions . .	1909	Cd. 4671	5	0
Decennial Supplement to Registrar-General's Report, Part I	1907	Cd. 2618	4	3
Decennial Supplement to Registrar-General's Report, Part II	1908	Cd. 2619	1	10
Report of Committee on the Census . .	1890	C. 6071	1	2
Report of Committee on the Income-Tax .	1906	H. of C. 365	2	7
Population Census — England and Wales: Summary Tables, 1901	1903	Cd. 1523	2	6
Population Census — England and Wales: General Report, 1901	1904	Cd. 2174	2	8
Population Census of 1911 and 1921. See Appendix III below.				
Census of Production, 1907		Cd. 6320	7	6
Wholesale and Retail Prices	1903	H. of C. 321	2	1

Wage Census of 1906. Textiles (Cd. 4545, 2*s.* 7*d.*), Clothing (Cd. 4844, 2*s.* 5*d.*), Building, etc. (Cd. 5086, 1*s.* 10*d.*), Public Utility Services (Cd. 5196, 1*s.* 9*d.*), Agriculture in 1907 (Cd. 5460, 8½*d.*), Engineering, etc. (Cd. 5814. 2*s.*). Railways in 1907 (Cd. 6053, 2*s.* 3*d.*).

Standard Time Rates. 1894 (C. 7567, ii., 1*s.* 3*d.*), 1906 (Cd. 3245, 7*d.*). 1909 (Cd. 4924, 6*d.*), 1913 (Cd. 7194, 7*d.*), 1920 (Cmd. 1253, 2*s.* 6*d.*).

Standard Piece Rates. 1894 (C. 7567, i., 1*s.*), 1900 (Cd. 144, 6*d.*).

Cost of Living in United Kingdom. 1913 (Cd. 6955, 4*s.* 11*d.*).

Cost of Living Committee, 1918 (Cd. 8980, 3*d.*).

Income-Tax Commission, 1919–20. Evidence Cmd. 288, 1 to 7.

Rents, Prices, Wages, etc., in Towns in the United Kingdom, 1908 (Cd. 3864, 6*s.*); in Germany, 1908 (Cd. 4032, 4*s.* 11*d.*); in France, 1909 (Cd. 4512, 4*s.* 1*d.*); in Belgium, 1910 (Cd. 5065, 2*s.* 2*d.*); in U.S.A., 1911 (Cd. 5609, 5*s.* 1*d.*).

APPENDIX III

The Population Census of 1911

The general form of the Householder's Schedule was unchanged, but a new question was introduced as follows :—

State, for each **Married Woman** entered on this Schedule, the number of :—

Completed years the **present** Marriage has lasted. **If less than one year write "under one."**	Children **born alive** to **present** Marriage. (If no children born alive write "None.")		
	Total Children Born Alive.	**Children still Living.**	**Children who have Died.**

The answers to this have led to a very interesting analysis in the Scotch Census (Vol. III, Cd. 7163), of the relation between the ages of parents and the number of children, and of the variation of the number of children when classified by the occupation of the father. The Irish report on the subject is rather fragmentary and the English is not yet published (July 1915).

The question as to occupation was altered and enlarged to read as follows :—

Personal Occupation.	Industry or Service with which worker is connected.
The reply should show the precise branch of Profession, Trade, Manufacture, etc. If engaged in any Trade or Manufacture, the particular kind of work done, and the Article made or Material worked or dealt in should be clearly indicated. **(See Instructions 1 to 8 and Examples on back of Schedule.)**	This question should generally be answered by stating the business carried on by the employer. If this is clearly shown in Col. 10 the question need not be answered here. **No entry needed for Domestic Servants in private employment.** If employed by a public body (Government, Municipal, etc.) state what body. **(See Instruction 9 and Examples on back of Schedule.)**

In the resulting tabulation (Vol. X, Part I, Cd. 7018) in the English Census, the occupations are first grouped as in the preceding Censuses, as described pp. 90 *seq.* above, and then according to the Industry or Service; in the latter the classification is according to the business of the employer, *e. g.* a carpenter or engineer permanently employed by a cotton firm is classified under Cotton, and a tram driver under Municipal Service. The differences shown are very instructive.

The question as to number of rooms in a tenement used to refer only to those of less than five rooms, but in 1911 was as follows :—

Write below the Number of Rooms in this Dwelling (House, Tenement, or Apartment). Count the kitchen as a room, but do not count scullery, landing, lobby, closet, bathroom; nor warehouse, office, shop.

One result of this improvement is to cause difficulty in comparison of overcrowding, since a considerable number of tenements of five rooms or more are now included among the overcrowded, whereas such tenements would have necessarily been ignored in previous records.

The general scheme of publication of the English Census was considerably improved. Each principal subject is dealt with in a separate volume, with an adequate introduction.

Besides the Preliminary Report the following volumes have been issued:—

A. *Population Statistics.*

Vol. I. Administrative Areas (Cd. 6258).
II. Poor Law and Registration Areas (Cd. 6259).
III. Parliamentary Areas (Cd. 6343).
IV. Ecclesiastical Areas (Cd. 6360).
V. Index to I–IV (Cd. 6576).
VI. Buildings and Population (Cd. 6577).

B. *Social Statistics.*

Vol. VII. Ages and Conditions as to Marriage (Cd. 6610).
VIII. Tenements (Cd. 6910).
IX. Birthplaces (Cd. 7017).
X. Parts I and II. Occupations and Industries (Cd. 7018, 7019).
Appendix. Rules for Classification, etc. (Cd. 7660).
XI. Infirmities (Cd. 7020).
XII. Language spoken in Wales and Monmouthshire (Cd. 6911).
XIII. Fertility of Marriage (Cd. 8678).
General Report (Cd. 8491).

In addition to this series, separate volumes are issued for each County, which bring together the tables relating to the County from the general volumes.

The increased detail and more elaborate analyses in the 1911 census have been made possible by the use of mechanical sorting and counting.

Population Census 1921

The Census was taken in June instead of in April owing to a railway strike. The populations of towns affected by holidays or the season are therefore not comparable with previous records. In estimating the population (as on p. 96 above) in non-censal years allowance must be made for this change of date.

The Householder's Schedule was altered in several particulars. The column Marriage was replaced by Marriage and Orphanhood, and in it persons divorced were to be marked D, and for children under fifteen it was to be stated

whether parents were alive or dead. The new question given on p. 217 above was dropped and a new question inserted asking for each married man, widower or widow, the number and age of all living children under sixteen years; this question will lead to tables relating to dependence. The address of each person's place of work was asked, in order to gain information about the day population of districts (as contrasted with the night population generally enumerated), and about the amount of daily travelling.

Preliminary Reports for England and Wales (Cmd. 1485), and Scotland (Cmd. 1473), showing population of each sex in each town, urban district, rural district, etc., but not those of civil parishes, were issued.

These are being followed by separate reports on each County—in Scotland on Edinburgh, Glasgow, Dundee and Aberdeen and thirty-three counties. Summary volumes will follow later. The more convenient method of the 1911 Census is thus given up, and since up to the end of 1922 only Part I for London, and thirteen Scottish parts have been issued, the general information for the country will be much delayed.

No Census was taken in Ireland.

The Census of Production

The Final Report of the Census of Production (Cd. 6320 of 1912) is a very important volume. It contains in great detail for all industries such information as is indicated on p. 133 above, and the introductory report discusses very carefully the relation between the results of this Census and the estimates of aggregate income, such as given on p. 173 above; * the new estimate for 1907 is given (p. 34 of the report) as "between 1,900 and 2,150 million pounds sterling."

* In fact, reference is made in the Report to this table as given in the first edition, before the report on small incomes was available.

The table given for Cotton above on p. 133 may now be extended so as to read—

INDUSTRIES OF THE UNITED KINGDOM, 1907.

Gross output	£1,765 millions
Materials used and work given out	1,053
Net output	712
Persons employed—	
Wage earners (excluding outworkers)	6,493,000
Salaried	492,000
Horse-power of engines at Factories, Mines, etc.	10,755,000

This table does not include agriculture, transport, commerce, or professional work. As to Agriculture, special reports were made by the Board of Agriculture and Fisheries (Cd. 6277) for Great Britain, and by the Department of Agriculture and Technical Instruction for Ireland,* giving an estimate for total agricultural production for the United Kingdom of about £210,000,000. The products of transport, commerce and professions are the subject of bold estimates in the Report.

RAILWAY STATISTICS

From January 1920 monthly statistics have been issued by the Ministry of Transport. (*Railway Statistics—Great Britain*, 2s. 6*d*., Monthly.)

These show for each company and for the country as a whole, number of tons carried, ton-miles, train-miles, etc., giving the kind of information discussed on p. 73 above. Ton-miles are shown separately for the three principal classes of traffic (general merchandise, coal, other minerals) and tons only for seventy-two classes, for each company. Passenger Statistics are also given. Receipts are shown in relation to each kind of traffic.

* Not numbered, but obtainable through Government publishers, price 2*d*., dated 1912.

APPENDIX IV

Revenue and Expenditure, 1908–1913

The details as given on p. 186 have not been published for subsequent years; but the following table has been arranged so as to continue as far as possible the same method of presentation, and to be used with p. 189 above. The first column gives 1908–9 as on p. 189. The second gives the same year modified so as to be strictly comparable with the following columns.

	1908–9.	1908–9.	1909–10.	1910–11.	1911–12.	1912–13.	1913–14.
	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000	£00,000
Revenue—							
Customs and Excise	62,9	62,9	61,4	73,2	72,1	71,5	75,0
Property and Income-tax . .	33,9	33,9	13,3*	61,9*	44,8	44,8	47,2
Estate, etc., duties .	18,4	18,4	21,8	25,5	25,4	25,2	27,4
Other taxes . .	10,4	10,4	8,7*	14,6*	12,7	13,3	13,4
Post Office† . .	4,8	4,2[a]	4,3	4,7	5,1	6,2	6,2
Crown lands, Suez Canal and miscellaneous . .	1,6	1,7[b]	1,7	1,7	1,8	1,9	2,1
Total net revenue .	132,0	131,5	111,2*	181,6*	161,9	162,9	171,3
Expenditure—							
National Debt .	31,4	28,0[c]	21,8	24,6	24,5	24,5	24,5
Naval and Military	56,8	59,0[c]	63,0	67,8	70,5	72,4	77,2
Education . .	15,4‡	17,4[d]	17,9	18,7	19,0	19,5	19,5
Insurance, Labour Exchange, Old Age Pensions . .	2,1	2,1	8,5	9,8	11,8	16,7	19,7
Civil List, etc. Other Civil Services .	10,3[f]	12,6[f]	13,6	13,7	14,4	14,5	14,2
Customs, Excise, Inland Revenue.	3,3	3,3	3,3	3,9	4,0	4,2	4,5
In Relief of Rates .	13,4	9,8[e]	9,6	9,9	9,6	9,7	9,7
Development and road Improvement . . .	—	—	—	1,4	1,7	1,2	1,4
Total net expenditure	132,7	132,2	137,5	149,8	155,5	162,7	170,7

[a] More expenditure charged to Post Office instead of to debt.
[b] Receipts for sundry loans transferred to debt. [c] As on p. 187.
[d] Whole vote Administration and Education. [e] Sums for police, etc., spent locally, left in other civil service. [f] Includes education other than elementary.

* Owing to postponement of Finance Bill, part of the money ordinarily due in 1909-10 was not paid till 1910–11.

† Revenue less Expenditure. ‡ Elementary.

INDEX

Abatements, 173, 175
Accuracy, 6–13, 69, 109
Addition, approximate, 8
Administrative counties, 87, 99
Ages, 36, 46, 98
Agricultural statistics, 134; wages, 147
Ancient counties, 99, 100
Approximation, 6–13
Argentina, trade of, 116
Arithmetical average, 17; errors of, 31–3
Attack rate, 108
Australia, trade of, 116
Austria, trade of, 116, 122
Averages, 15–26, 54, 67, 108, 110; accuracy of, 27–34; calculation of, 19, 20; descriptive, 20; errors of, 29–31; in sampling, 58; weighted, 17

Belgium, trade of, 116, 122
Bertillon, M., 102
Biassed errors, 29, 31, 32
Birth-rate, 2, 67, 102 *sqq.*
Blank forms, 81
Boroughs, 87, 100, 101
Brabrook, Sir E. W., 166
Brazil, trade of, 116
Budgets, working-class, 58, 166–70
Building, numbers, 91; unemployment, 153, 158; wages, 143, 147
Bulgaria, trade of, 116
Bullion, imports and exports of, 113, 114, 116, 117

Canada, trade of, 115, 116
Capital, 182–4
Case fatality rate, 108
Census, population, 85–101, 213–15; production, 133, 215–16; wages, 142
Changes in wages, 141–8
Chapman, Prof., 152
Chile, trade of, 116
China, trade of, 116
Civil parishes, 87, 99, 101
Classes, statistical, 1
Clothing trade, wages, 149
Coal, exports, 121; prices, 128–9; production, 133
Coal miners, numbers, 91; wages, 143–5
Collection of statistics, 80
Comparison, criticism of, 68; error in, 32; nature of, 11
Co-operative societies, 166
Corrected death-rates, 105–6
Correction factor, 105–6
Costa Rica, trade of, 116
Cotton, exports, 121; numbers employed, 92; prices, 128–9; production, 133; wages, 22, 149
Counties, 87, 99, 100
County boroughs, 87, 100
Craigie, Major, 134
Crawford, R. F., 134
Criticism of statistics, 64–70
Customs, 185–6, 188–9, 222

Death-rates, 2, 65, 102 *sqq.*
Deciles, 62, 199
Definition, importance of, 3, 57, 64, 108, 132
Denmark, trade of, 116
Density of population, 92–3
Diagrams, 22, 35–49, 54, 109; circular, 47; cumulative, 37
Division, approximate, 10
Dudfield, Dr., 102

Earned income, 180–1
Ecclesiastical parishes, 87, 101
Economist, The, 126
Education, expenditure on, 186–7, 189, 194, 222
Emigration, 95, 96
Employment, 151–60

Engineering, unemployment, 153, 156, 157–9; wages, 143, 147
England and Wales, census of, 86; population, 89
Errors, absolute, 28–34; biassed, 29; meaning and nature of, 7, 8, 28; percentage, 28; relative, 28–32; unbiassed, 29
Estate duty statistics, 182–5, 189, 190, 217
Excise, 185, 188, 189, 222
Exemptions, 173, 174, 177, 180
Exports, 111–22; iron and steel, 45; prices, 126–8, 132; worsted goods, 76

Finland, trade of, 116
Fish, 134
Fluctuations, 41–3, 69
Food, imports and exports, 121
Foreign trade, 111–22
France, trade of, 116, 122; shipping of, 123
Frequency curves, 22
Friendly societies, 165

Germany, trade of, 116, 122; shipping of, 123
Greece, trade of, 116
Groups, statistical, 1, 35, 36

Hallsworth, H. M., 152
Hayward, T. E., 97
Holland, trade of, 116, 122
Homogeneity, 65, 67, 75, 79, 90
Hooker, R. H., 134
Hull, 88

Immigration, 95, 96
Imports, 119–22; prices, 126–31, 132
Income-tax statistics, 173–9, 181, 184, 189, 222
Index-numbers, prices, 76, 128–32; wages, 146–7, 180; incomes, 180
India, trade of, 116
Indices, 66
Infant mortality, 104, 105, 107
Inhabited house duty, 190–2
Inland Revenue, 185–6
International trade, 116
Ireland, census, 86; population, 89; trade, 114
Iron and steel, exports, 45, 121; prices, 128–9; production, 133, 135–6; manufacture, wages in, 143–5
Italy, trade of, 116; shipping of, 123

Japan, trade of, 116; shipping of, 123
Jute, prices, 128–9

Labour Exchanges, 152; expenditure on, 184, 222
Labour Gazette, 126, 142, 152–3
London, 27, 89, 101
Longstaff, Dr., 110

Mallet, B., 182
Marriage-rates, 2, 102, 104
Mean, 21, 22
"Meat and Milk Committee," R.S.S., 134
Median, 22, 24, 33, 39, 62
Medical officers of health, 102, 107
Merchant navies, 123
Metropolitan boroughs, 27, 101
Mexico, trade of, 116
Middlesbrough, 93
Mineral statistics, 134
Miners' wages, 143–5
Mixture for sampling, 56
Mode, 21, 22, 32, 39
Morbidity rates, 108
Multiplication, approximate, 9
Municipal boroughs, 87, 100

National income, 171–84
,, insurance, 170; expenditure on, 194, 222
Natural increase, 95, 97
Newsholme, Dr., 102
Normal week, 137
Norway, trade of, 116; shipping of, 123

Occupations, numbers in, 90–2
Old age pensions, 170; expenditure on, 189, 194, 222
Overcrowding, 80, 81, 94

Parishes, 87, 99
Percentages, 67; mistakes in, 14; method of, 11
Peru, trade of, 116
Piece-rates, 138–9
Poor relief, expenditure on, 194

Population, 6-7; census, 85-101, 217-20; England and Wales, 89, 95; Ireland, 89; London, 27, 89; Scotland, 89; United Kingdom, 89, 96
Portugal, trade of, 116
Predominant rate, 21
Prices, 126–32
Printers, unemployment, 153, 158, 160
Production, 133–6, 220–21
Published statistics, rules for using, 64–70

QUARTILES, 24, 39, 62

RAILWAY statistics, 72–4, 124, 221
Random selection, 57
Rates, as averages, 16, 67
,, *see* birth-, death-, marriage-
,, statistics of, 193–5
Ratio, nature of, 11; error in, 32
Raw material, imports and exports, 121
Real wages, 148, 150
Registrar-general, 86, 102 *sqq.*
Registration, counties, 87, 99; districts, 87, 99
Retail prices, 127
Rew, R. H., 134
Rural districts, 87, 89, 94, 100, 101
,, population, 89
Russia, trade of, 116, 122

SAMPLING, 56–63
Sauerbeck, A., index-numbers computed by, 126, 128, 130, 131
Savings, working-class, 166
Schloss, D. F., 139
Scotland, census, 86; population, 89
Series, statistical, 2, 35, 41, 43, 69
Shipbuilding, unemployment, 153, 156, 158–9
Shipping, 122–5
,, tonnage, note on, 124–5
Ships, built, 135; exported, 121
Silver, prices, 128–9
Sliding-scales, 141 (*note*)
Smoothing, 41, 96
South Africa, trade of, 116
Spain, trade of, 116; shipping of, 123

Square root, approximate, 10
Stamp, Sir Josiah, 182
Standard death-rate, 105–6
Statist, The, 126
Strikes, statistics of, 160–3
Strutt, H. C., 182
Subtraction, approximate, 8
Sugar, prices, 128–9
Super-tax, 182
Sweden, trade, 116; shipping, 123
Switzerland, trade of, 116, 122

TABULATION, 50–5, 108
Tax statistics, 185–95, 222
Tea, prices, 128–9
Textile manufactures, numbers occupied, 91; wages, 143–5, 147
Time wages, 138, 140
Tinplate production, 133
Ton-miles, 73
Tonnage, shipping, 122–5
Trade-union statistics, 163–4
Transhipment, 113
Trend, 41–3, 69

UNBIASSED errors, 29, 31, 32
Unearned income, 180–1
Unemployment, 4, 151–60
U.S.A., trade of, 115–16; shipping of, 123
Urban districts, 87, 89, 94, 100, 101
,, population, 88, 89
Uruguay, trade of, 116

VITAL statistics, 102–10

WAGES, 67, 77; index-numbers of, 146–7; statistics of, 22, 51, 137–50
Wages and incomes, changes compared, 180
Waters, A. C., 97
Weights, weighted averages, weighted totals, 17, 18, 32, 76, 77
Wheat, prices, 128–9
Wood, G. H., 145, 148
Wool, exports, 121; numbers employed in manufacture, 92; prices, 128; statistics, 136
Worsted manufactures, exports, 76

YORKSHIRE, 93

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